



# Chemical diffusion in silicate melts

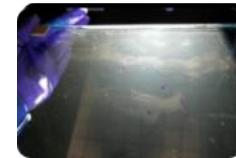
Emmanuelle Gouillart, Ekaterina Burov  
Saint-Gobain Research Paris  
Joint Unit CNRS / Saint-Gobain



## Improvement of glass products properties



500 staff working on industrial processes and materials for construction & industry: glass, mineral wools, gypsum, mortars, composites..



2050  
NET ZERO CARBON

## Decarbonation of glass processes



# Acknowledgements

---

SVI joint unit : Katia Burov, MH Chopinet, H Montigaud, F Pigeonneau

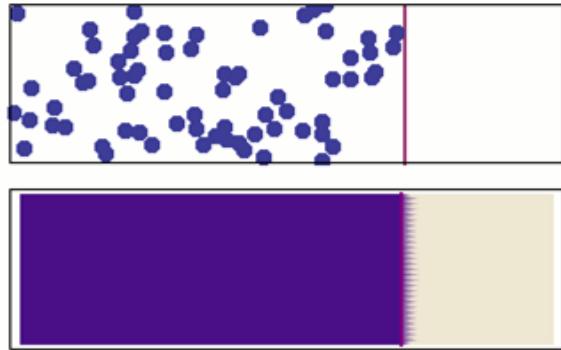
Saint-Gobain Research Paris: C. Joussemae, S. di Pierro, S. Papin

PhD Students: W. Woelffel, C. Claireaux, M. Ficheux, M. Jacquemin, S. Ben Khemis, B. Bouteille, JT Fonné, F. Yoshizawa

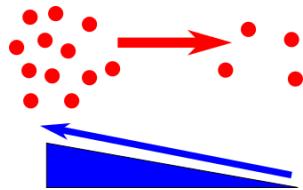
Collaborations: MAGI project, M. Toplis, M. Roskosz, L. Cormier, P. Simon, C. Bessada, E. Véron, M. Salanne, S. Schuller, H. Pablo, D. Vandembroucq.



# Chemical diffusion



$$\mathbf{j} = -D \nabla C$$



$$\frac{\partial C}{\partial t} = D \Delta C$$

Zhang, Y., & Gan, T. (2022). Diffusion in melts and magmas. *Reviews in Mineralogy and Geochemistry*, 87(1), 283-337.

Diffusion couple

Initial ( $t=0$ )



$t=t$



Sorption  
(constant surface)



Mineral dissolution



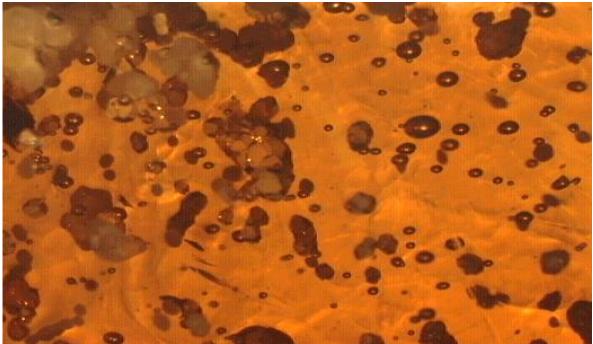
Instantaneous  
source  
(thin film)



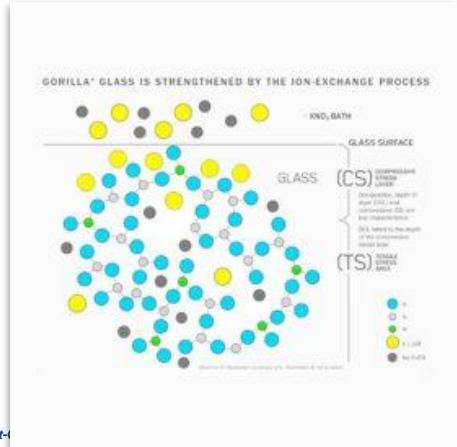
# Consequences and applications of molecular diffusion in silicate melts



## Glass melting: batch & stones



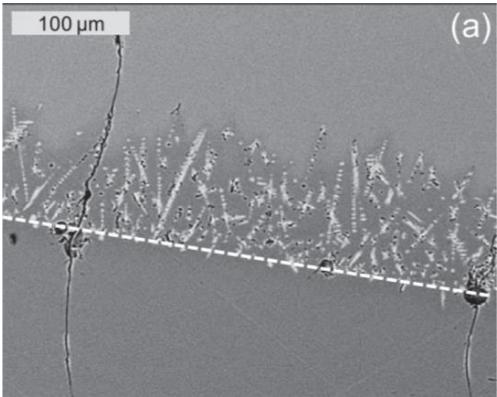
## Ionic exchange (display)



## Refractory corrosion



## Crystallization



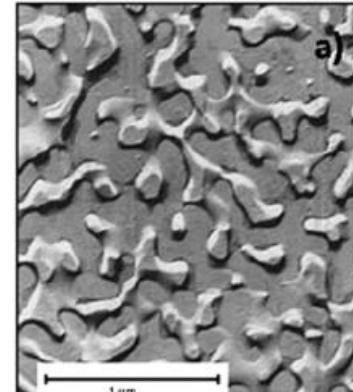
SAINT-GOBAIN RESEARCH  
PARIS

Pablo et al  
JNCS 2019.

## Volatile diffusion & volcanic eruption



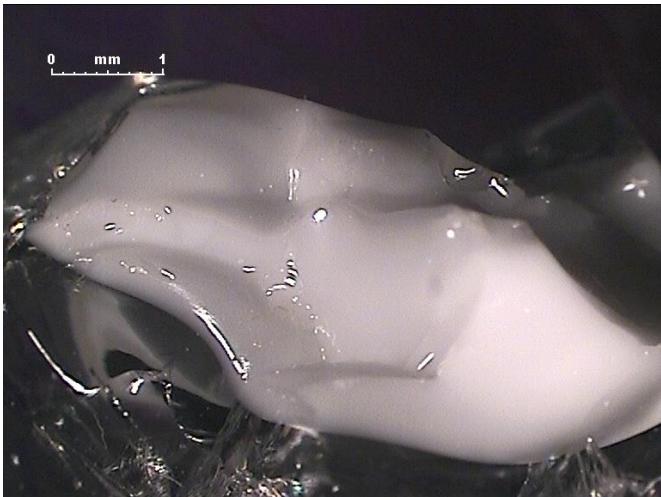
## Phase separation



SAINT-GOBAIN

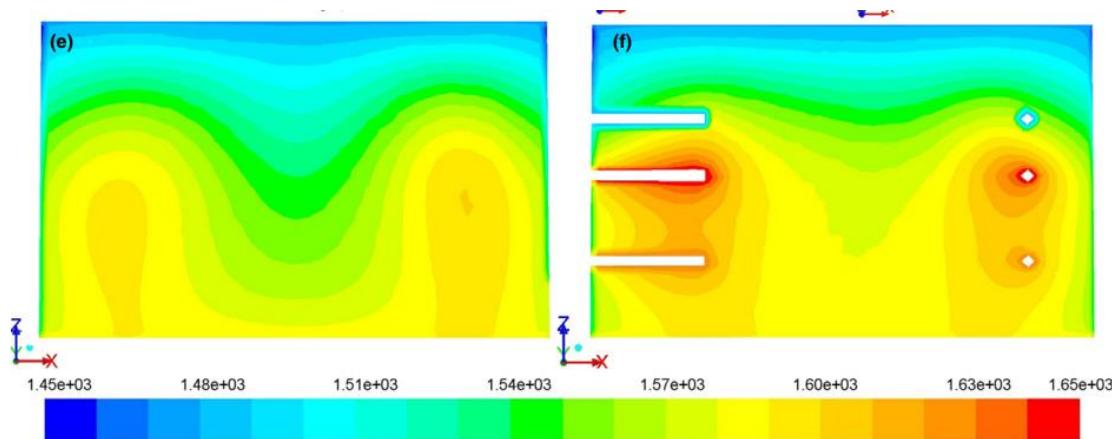
# Why is diffusion important for sustainable glass?

More cullet → more impurities to dissolve



Courtesy of S. Di Pierro, SGR Paris

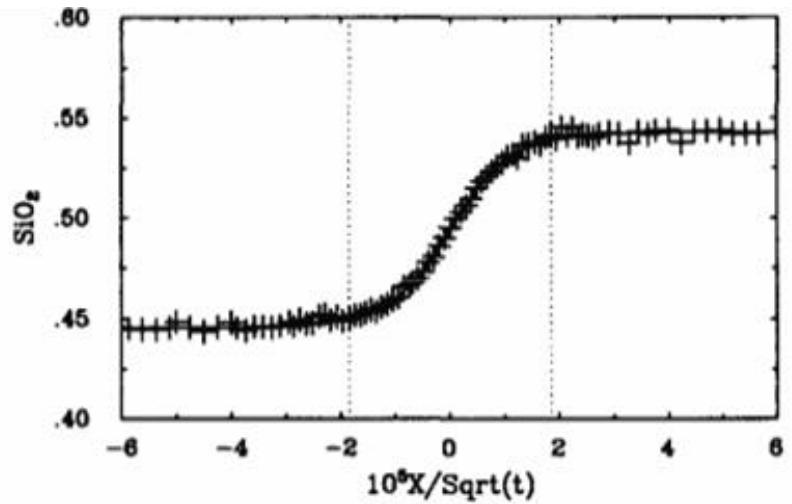
More corrosion of refractories with electric melting



Li, Hailong, et al. "3D simulation of borosilicate glass all-electric melting furnaces." *Journal of the American Ceramic Society* 97.1 (2014): 141-149.

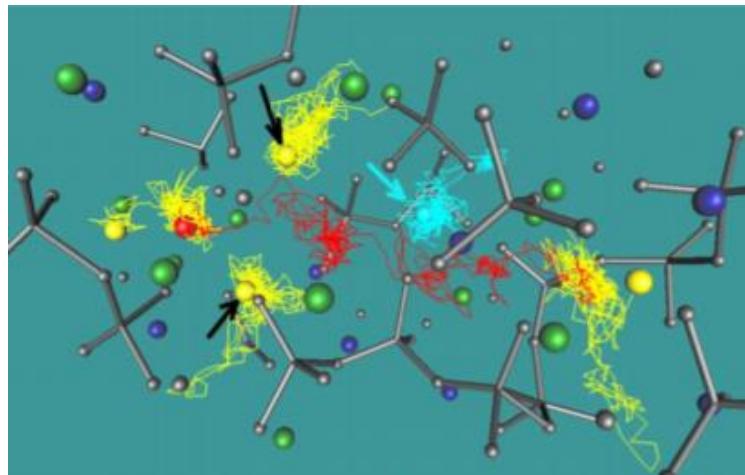
# Measuring diffusion data

Concentration gradients (chemical concentrations, isotopes...)



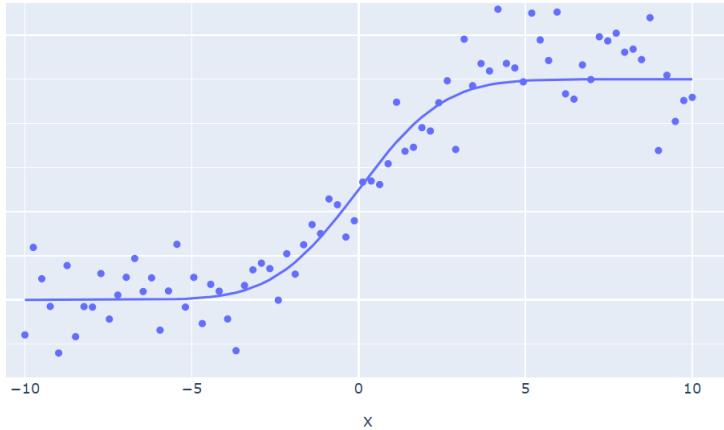
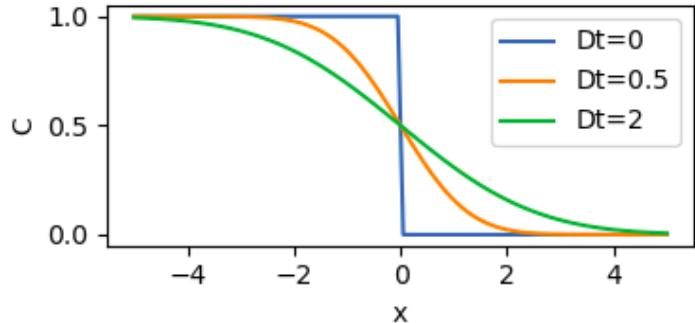
Liang, Yan, Frank M. Richter, and E. Bruce Watson. *Geochimica et Cosmochimica Acta* 60.24 (1996): 5021-5035.

Analysis of trajectories in MD  
Einstein formula (fluctuation-dissipation relation)



Tilocca, Antonio. *The Journal of chemical physics* 133.1 (2010): 014701.

# Fitting experimental diffusion profiles



Fit known parametric laws through noisy experimental points.

$$n(x, t) = n_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right).$$

Importance of :

- spatial resolution and number of points
- experimental noise

# Some orders of magnitudes

Diffusivity of gaseous molecules in air ?  $\sim 10^{-5} \text{ m}^2.\text{s}^{-1}$  (in 1s:  $l \sim \text{a few mms}$ )

Diffusivity of dissolved species in water ?  $\sim 10^{-9} \text{ m}^2.\text{s}^{-1}$

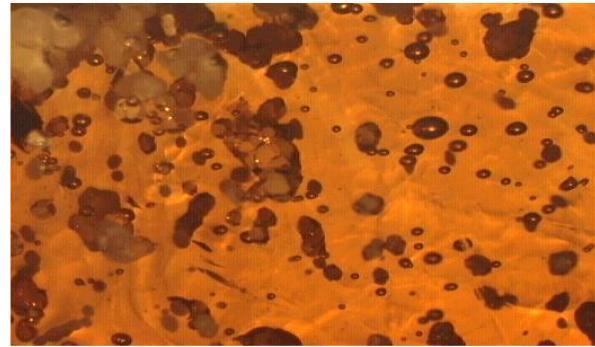
Diffusivity of Si in soda-lime silica melt at 1200°C :  $\sim 10^{-12} \text{ m}^2.\text{s}^{-1}$

in 1s:  $l \sim 1 \text{ micron}$

in 1 hour:  $l \sim 60 \text{ microns}$

in 24 hours:  $l \sim 300 \text{ microns}$

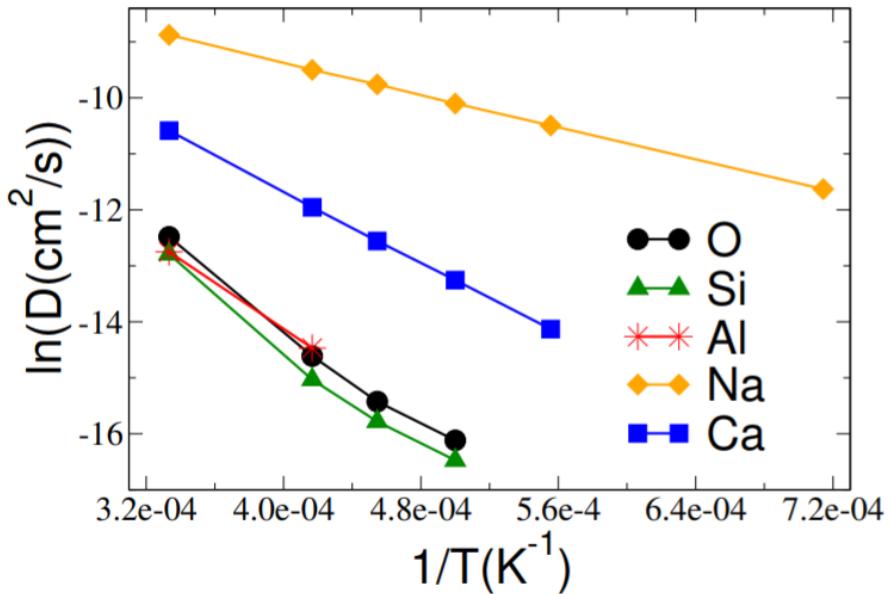
Fortunately, convection can accelerate diffusion. **Peclet number** :  $Pe = Ul / D$



Shrinking rate of particles  $\sim Pe^{1/3}$  Assunção, M., M. Vynnycky, and K. M. Moroney. "On the dissolution of a solid spherical particle." *Physics of Fluids* 35.5 (2023).

# Values of diffusivities in silicate melts - influence of species

Review : Zhang, Y., Ni, H., & Chen, Y. (2010). Diffusion data in silicate melts. *Reviews in Mineralogy and Geochemistry*, 72(1), 311-408.

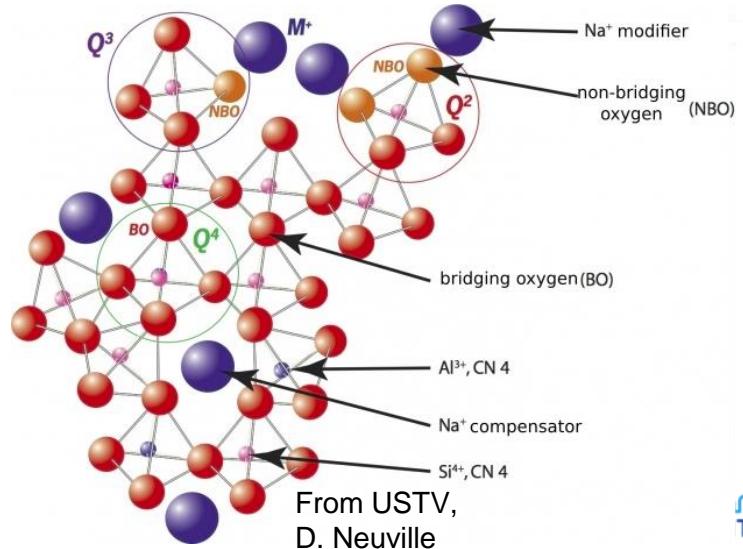


Col with Mathieu's Salanne's team: Serva, Alessandra, et al. "Structural and dynamic properties of soda-lime-silica in the liquid phase." *The Journal of Chemical Physics* 153.21 (2020): 214505.

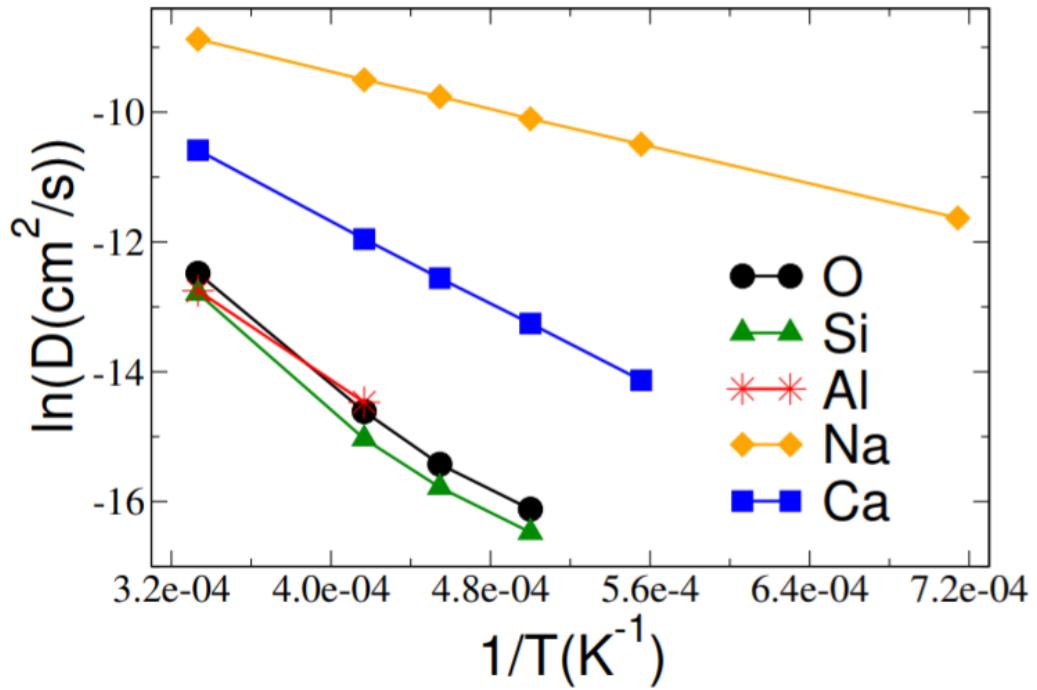
D depends on **strength** and **number** of chemical bonds (i.e. on silicate structure)

$D(\text{network modifiers}) > D(\text{network formers})$

$D(\text{monovalent alkali ions}) > D(\text{divalent alkali-earth ions})$

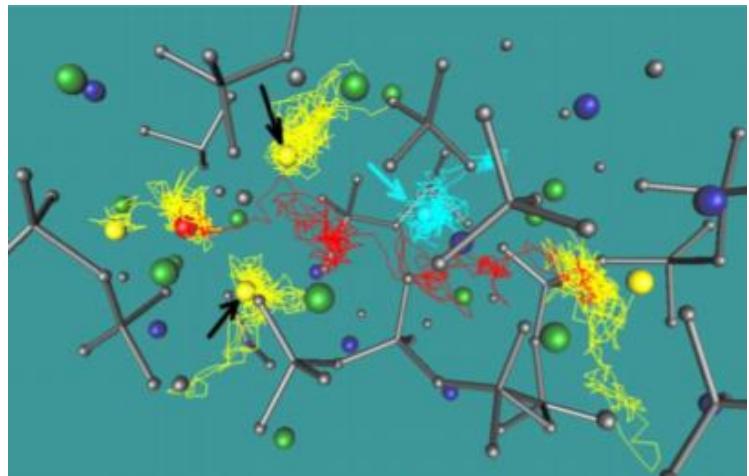


# Values of diffusivities in silicate melts - influence of temperature



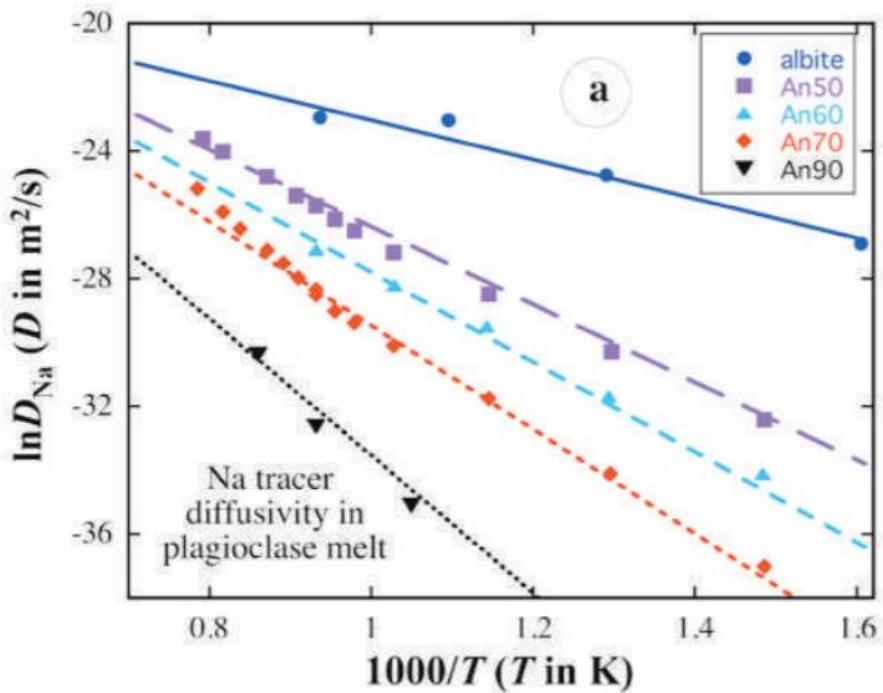
Serva, Alessandra, et al. "Structural and dynamic properties of soda-lime-silica in the liquid phase." *The Journal of Chemical Physics* 153.21 (2020): 214505. Coll. M. Salanne, MAGI project

Arrhenian behaviour  
Activation energy related to chemical bonds



Tilocca, Antonio. *The Journal of chemical physics* 133.1 (2010): 014701.

# Values of diffusivities in silicate melts - influence of composition



Self-diffusion of sodium in various silicate melts

Zhang, Y., Ni, H., & Chen, Y. (2010). Diffusion data in silicate melts. *Reviews in Mineralogy and Geochemistry*, 72(1), 311-408.

More sodium → larger D of sodium.

Qualitative trend: D increases when viscosity decreases.

$$D_{\text{Na TD}}^{\text{plag melt}} = \exp \left[ -16.87 + 5.318 X_{\text{An}} - \frac{(6158 + 5769 X_{\text{An}} + 12480 X_{\text{An}}^2)}{T} \right]$$

## Einstein relations

$$D_i = M_i kT \gamma_i \quad \text{mobility} \quad M_i = v/F$$

The diffusion of different kinds of species can be investigated through different physical quantities.

Charged particles: Nerst-Einstein relation

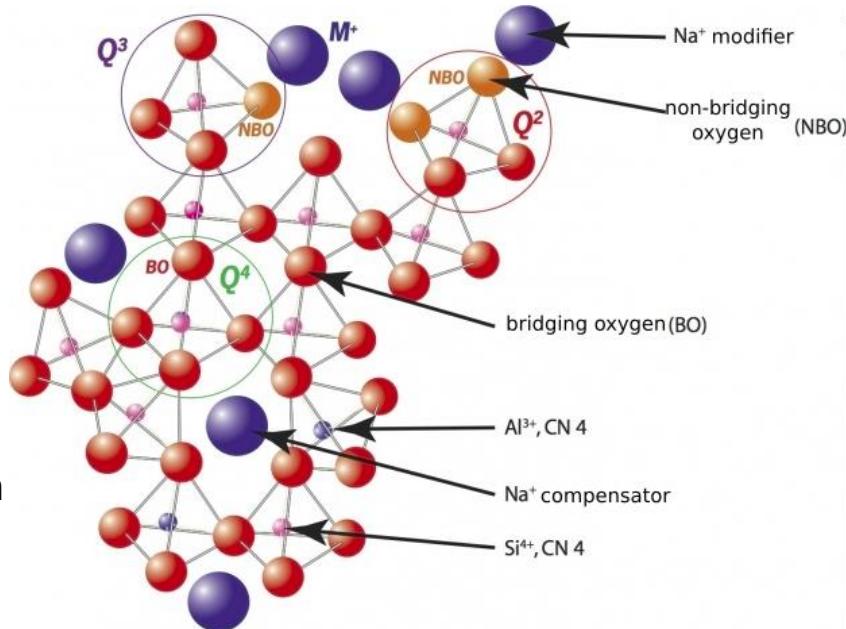
→ **network modifiers**

$$D = \frac{\mu_q kT}{q} \quad q: \text{charge of ions}$$

Viscous liquids: Stokes-Einstein and Eyring relation

→ **network formers**

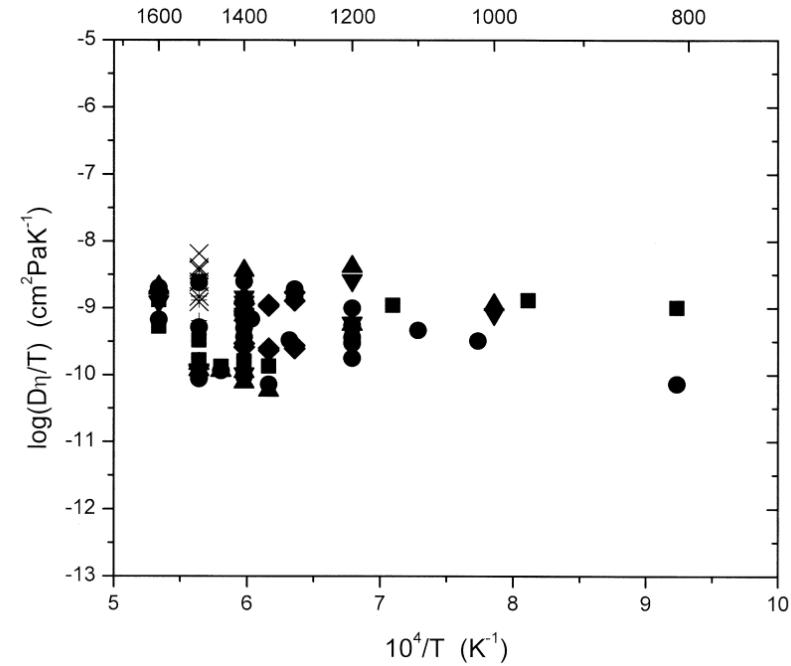
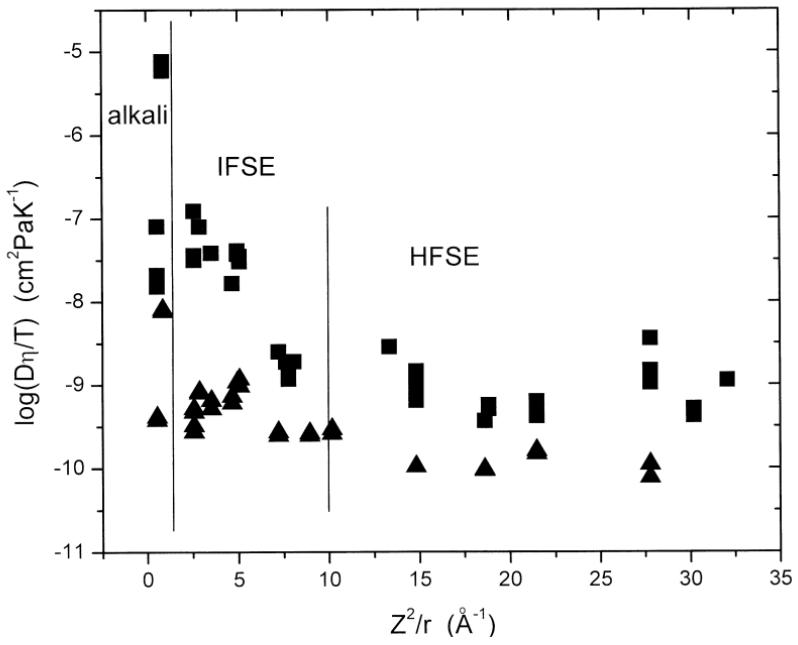
$$D = \frac{k_B T}{6\pi \eta r}. \quad D = \frac{kT}{2\eta r} \quad \eta: \text{viscosity} \quad r: \text{radius of particle}$$



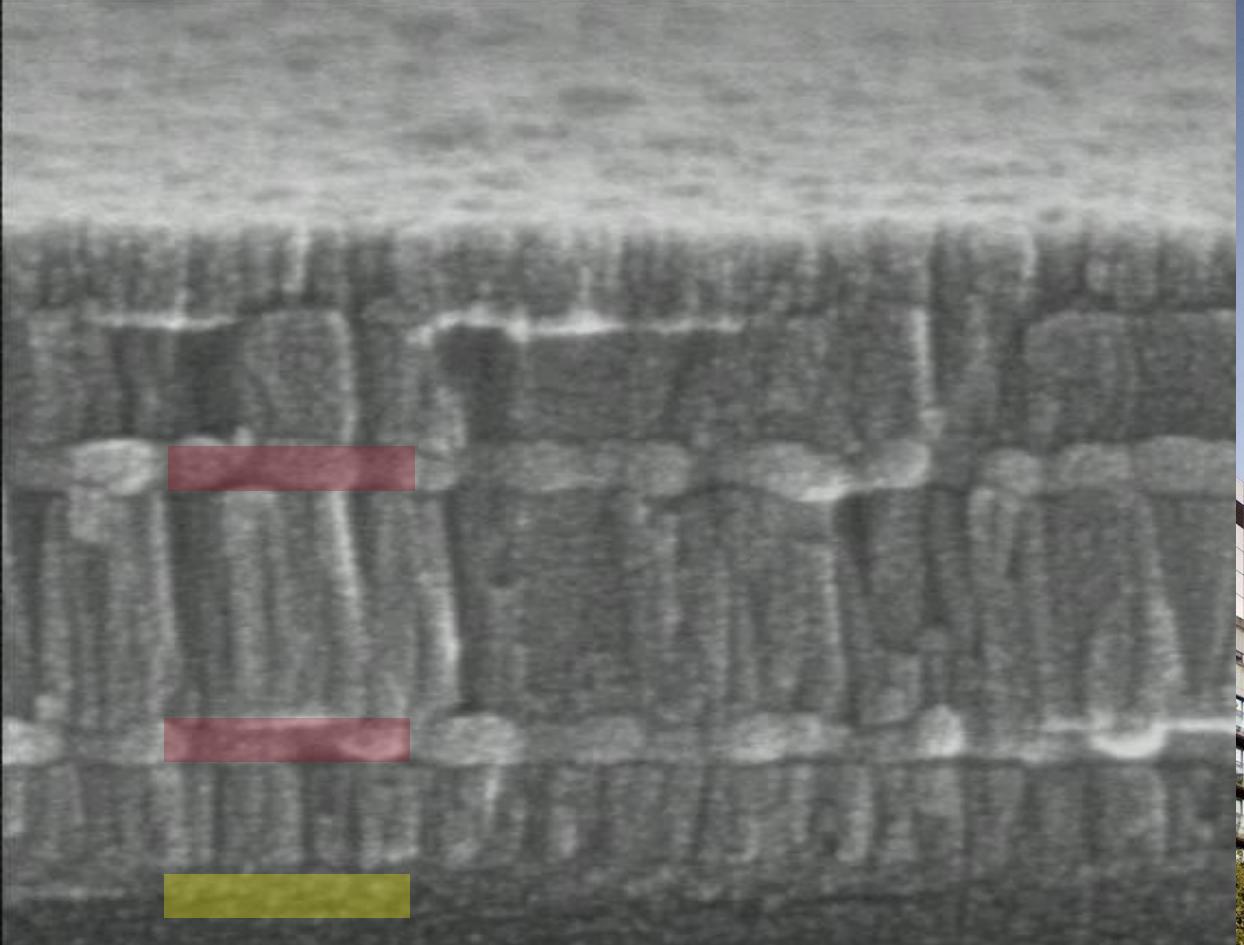
from USTV website  
D. Neuville

# Relating diffusion and viscosity

Viscosity and diffusion



Empirical models relating viscosity and tracer diffusion in magmatic silicate melts, Mungall GCA 2002.

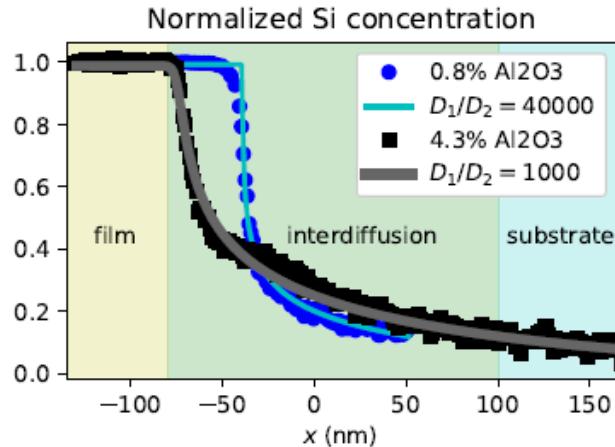


x200000 200nm  
#0 ECH. NON BOMBE  
512 x 512

15kV  
\*SGR\* M42  
1-



# Diffusive dissolution of thin film and multicomponent effects



$$\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C)$$

D often considered constant, but sometimes this approximation cannot be used

silica thin film

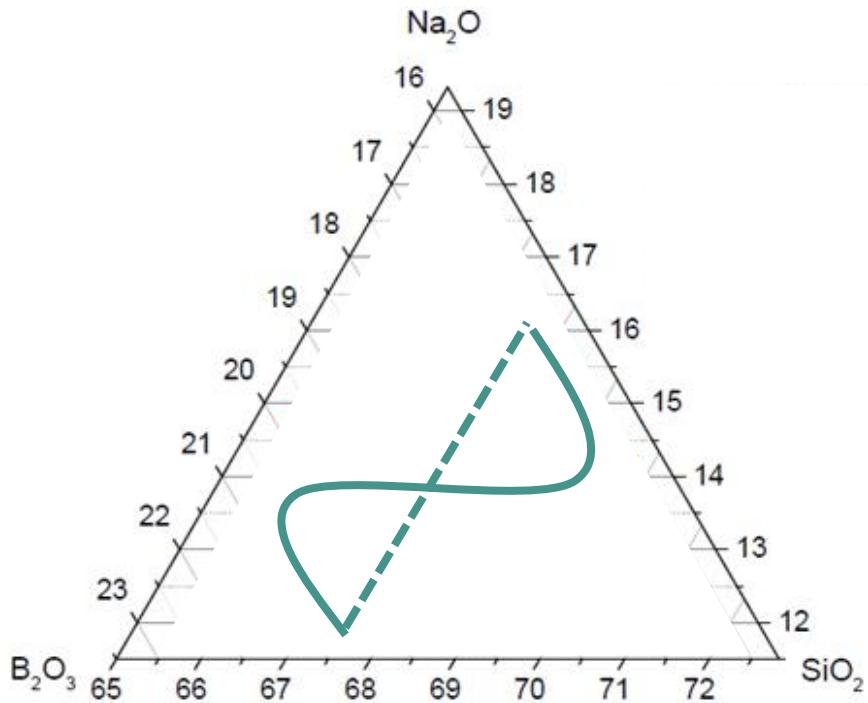
smaller D

soda-lime glass

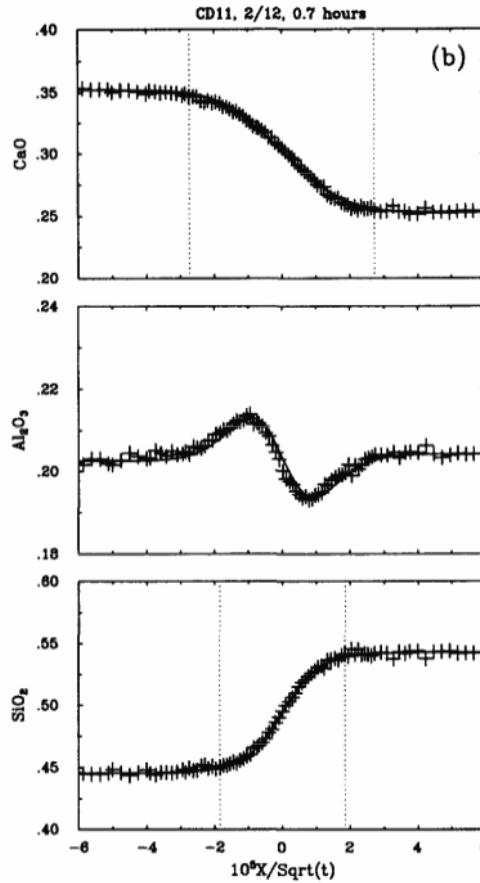
larger D

PhD JT Fonné ;  
Fonné et al., JACS  
2017, JACS2018  
PhD S. Ben  
Khemis, coll. L  
Cormier, D.  
Vandembroucq

# Multicomponent diffusion



From Pablo et al JNCS 2019,  
coll. with the team of Sophie Schuller.



uphill diffusion

Liang, Yan, Frank M. Richter,  
and E. Bruce Watson.  
*Geochimica et Cosmochimica Acta* 60.24 (1996): 5021-5035.

# Diffusion matrix: how to describe multicomponent diffusion



$$\mathbf{j}_i(\mathbf{x}) = - \sum_k D_{ik} \nabla C_k(\mathbf{x})$$

$$\frac{\partial C_i}{\partial t} = \sum_k D_{ik} \Delta C_k(\mathbf{x})$$

$$\frac{\partial \mathbf{C}}{\partial t} = \mathbf{D} \Delta \mathbf{C}(\mathbf{x})$$

Measured in several ternary systems,  
mostly in [geosciences](#)

[Liang et al., 1996], [Richter et al., 1998] :  
CaO/MgO – Al2O3 – SiO2.

[Review by Y. Liang in 2010](#)

Also used in multicomponent metallic alloys.

More rigorously: thermodynamic formulation  
with activity gradient (Onsager).

[Few studies in industrial systems.](#)

$$\frac{\partial}{\partial t} \begin{pmatrix} C_{\text{Na}} \\ C_{\text{Ca}} \\ C_{\text{Al}} \\ C_{\text{Si}} \end{pmatrix} = \boxed{\begin{pmatrix} D_{\text{Na},\text{Na}} & D_{\text{Na},\text{Ca}} & D_{\text{Na},\text{Al}} & D_{\text{Na},\text{Si}} \\ D_{\text{Ca},\text{Na}} & D_{\text{Ca},\text{Ca}} & D_{\text{Ca},\text{Al}} & D_{\text{Ca},\text{Si}} \\ D_{\text{Al},\text{Na}} & D_{\text{Al},\text{Ca}} & D_{\text{Al},\text{Al}} & D_{\text{Al},\text{Si}} \\ D_{\text{Si},\text{Na}} & D_{\text{Si},\text{Ca}} & D_{\text{Si},\text{Al}} & D_{\text{Si},\text{Si}} \end{pmatrix}} \Delta \begin{pmatrix} C_{\text{Na}} \\ C_{\text{Ca}} \\ C_{\text{Al}} \\ C_{\text{Si}} \end{pmatrix}$$

# Example: multicomponent diffusion in a quaternary system

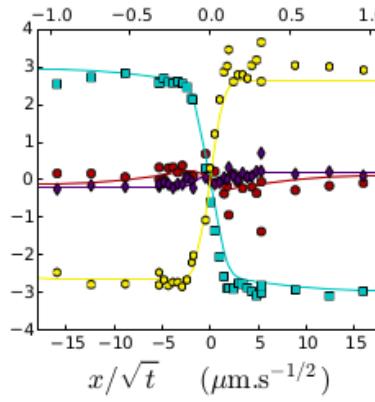
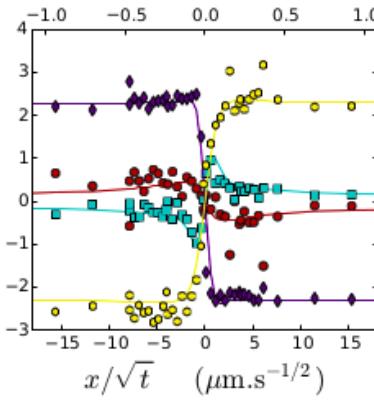
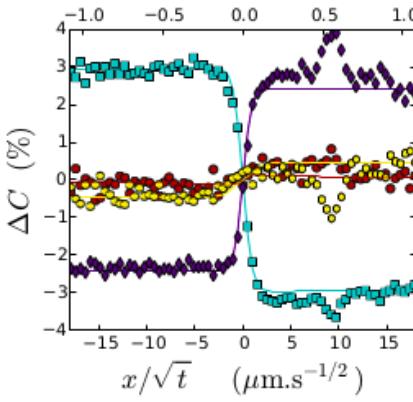
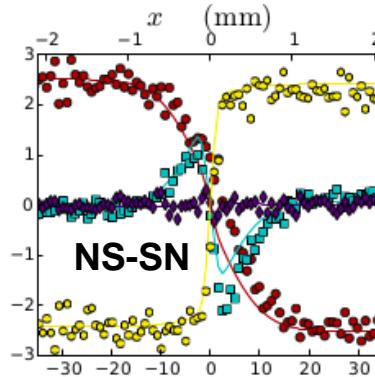
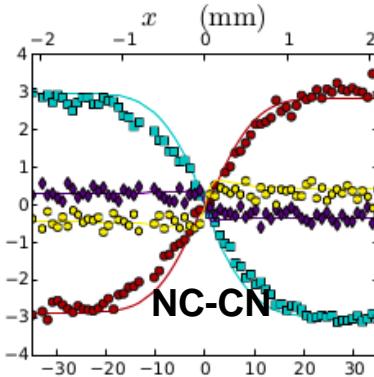
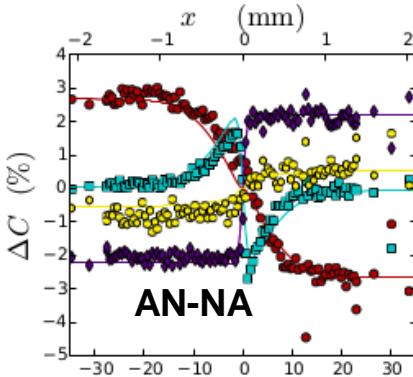
•Na<sub>2</sub>O

■CaO

◆Al<sub>2</sub>O<sub>3</sub>

●SiO<sub>2</sub>

PhD C. Claireaux

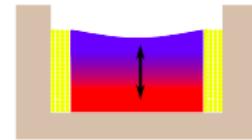


CS-SC

$t = 0$



$t$



6 diffusion-couple experiments

Claireaux, Corinne, et al. "Atomic mobility in calcium and sodium aluminosilicate melts at 1200 C." *Geochimica et Cosmochimica Acta* 192 (2016): 235-247.

**multidiff** open-source code:  
diffusion matrix, **eigenvalues & eigenvectors**

# Multidiff: an open-source package for fitting multicomponent diffusion data



← → C ⌂ pythonhosted.org/multidiff/auto\_examples/index.html

Présences site Séminaires de 11h... Livelink - Direction...

[multidiff 0.1 documentation »](#)

[Previous topic](#)

[Installation](#)

[Next topic](#)

[Fitting multidiffusion profiles  
for three components](#)

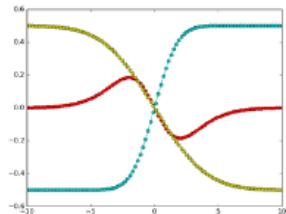
[This Page](#)

[Show Source](#)

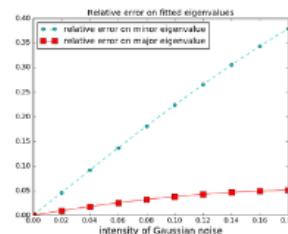
[Quick search](#)

Go

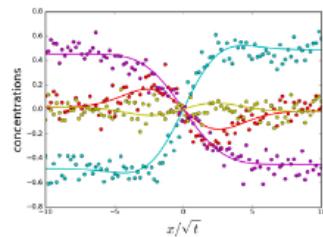
## Examples



Fitting multidiffusion  
profiles for three  
components



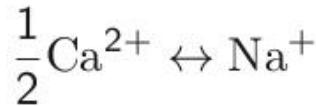
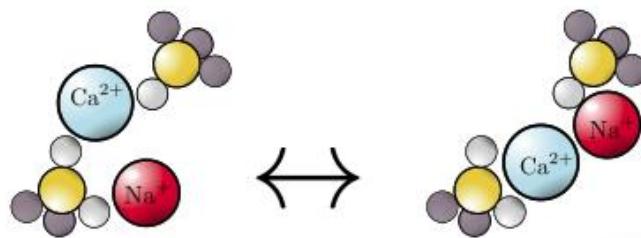
Effect of measurement  
noise on accuracy of fit



Effect of initialization on  
accuracy of fit

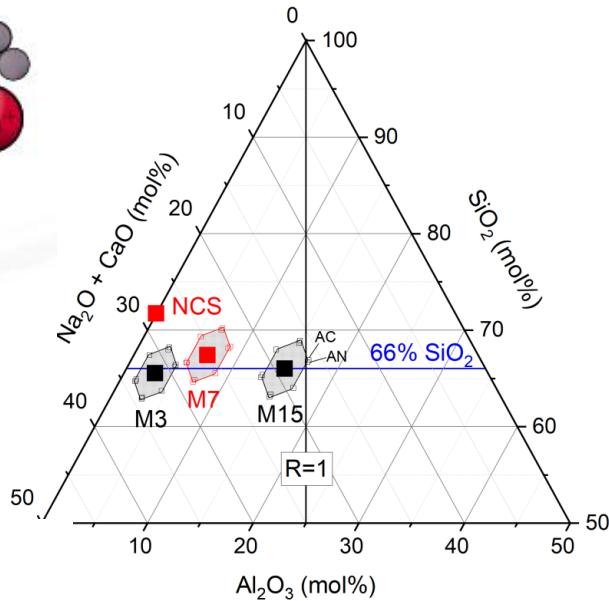
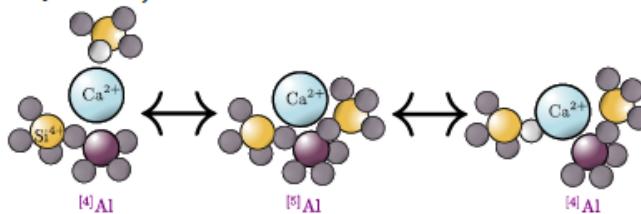
## Dominant eigenvector

$$\Delta \begin{pmatrix} C_{\text{Na}_2\text{O}} \\ C_{\text{CaO}} \\ C_{\text{Al}_2\text{O}_3} \\ C_{\text{SiO}_2} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$



**Other eigenvectors:** involve network formers, smaller diffusivity value

**Second eigenvector** (52x less frequent)



PhD M. Jacquemin,  
ANR MAGI, CEMHTI  
P. Simon, C. Bessada,  
E. Burov

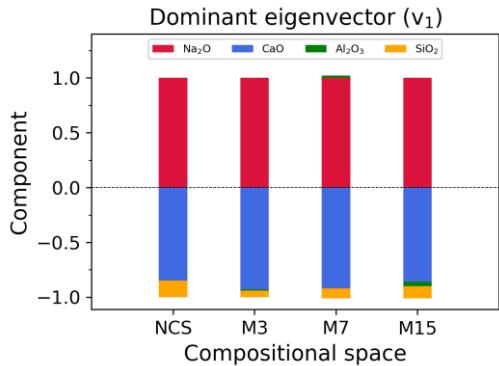
# Diffusion pathways in Peralkaline compositions area

PhD Maxime Jacquemin, MAGI project

NCS : Trial&Spera, 1994

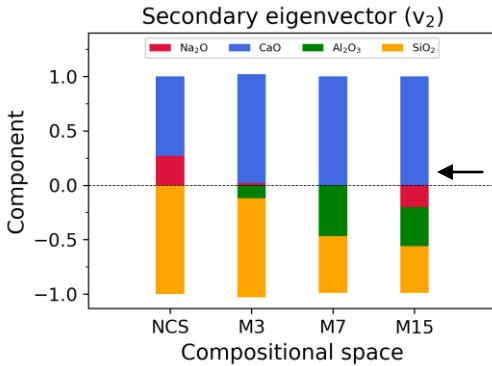
M7 : Claireaux et al., Geochim. Cosmochim. Acta 2016, 2019

## Eigenvectors ( $\vec{v}_i$ )



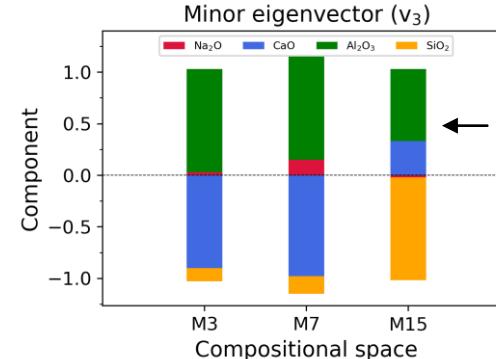
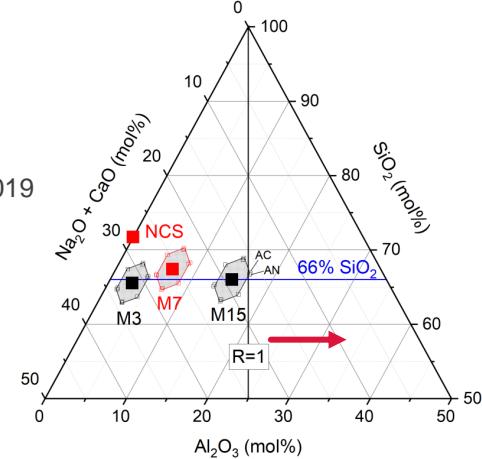
$v_1 : \text{Na} \leftrightarrow \text{Ca}$

- dominant vector is always Na  $\leftrightarrow$  Ca
- Independent of Al<sub>2</sub>O<sub>3</sub> concentration and the structures role of Na and Ca

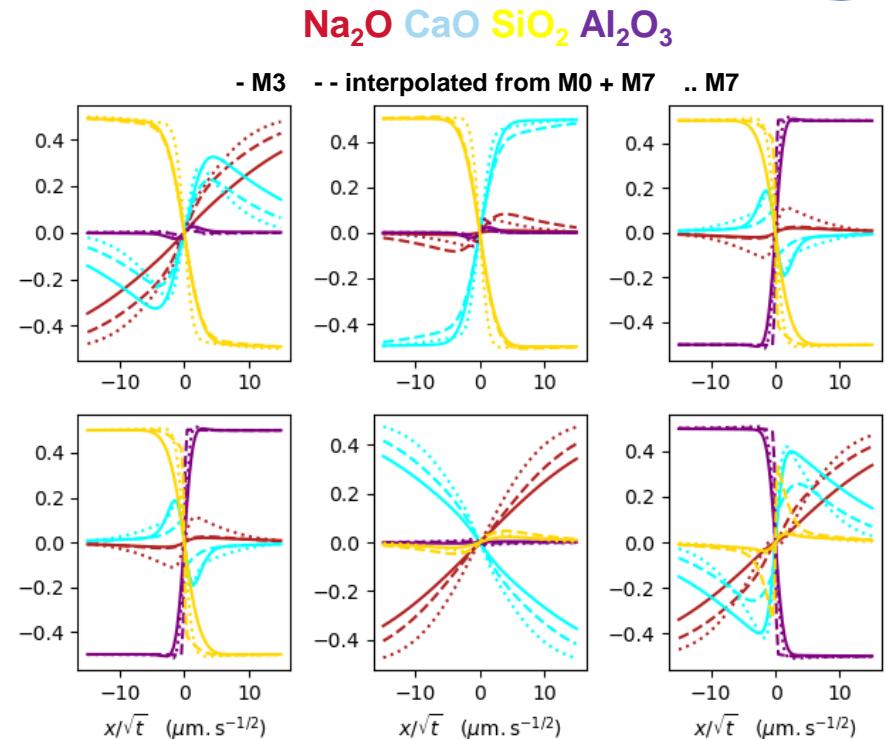
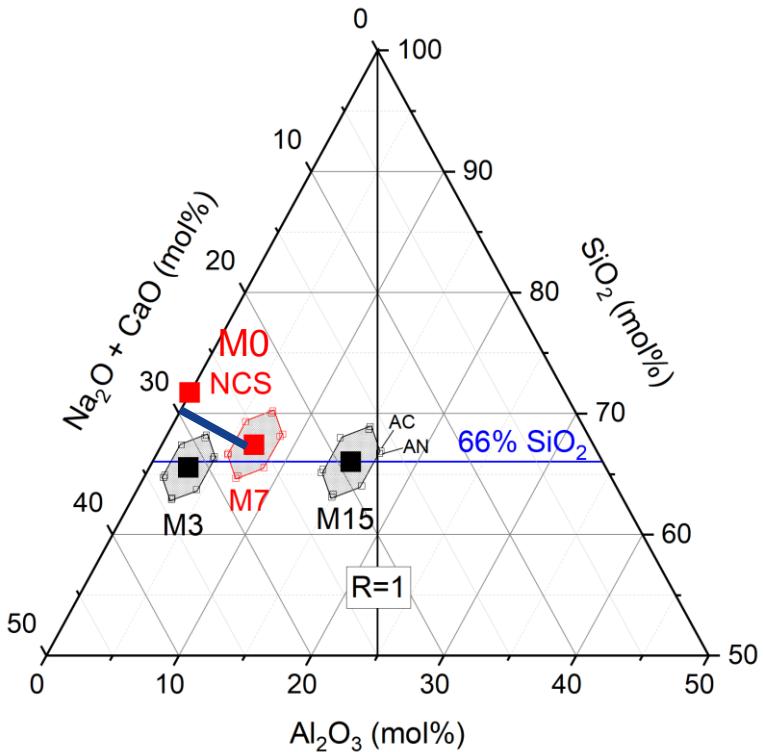


$v_2 \text{ et } v_3 : \text{Formers} \leftrightarrow \text{Ca}$

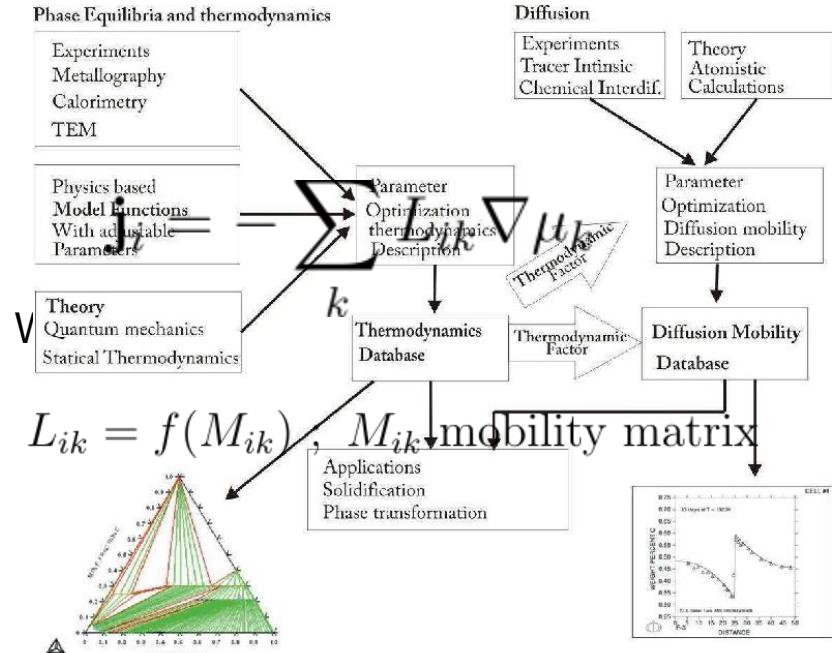
- Reaction evolution with Al<sub>2</sub>O<sub>3</sub> concentration
- Al is associated to the Na or Ca near R=1



# Can we interpolate diffusion matrices ?



# Multicomponent diffusion and thermodynamics



Hypothesis: model of mobility matrix  $M$  with composition.

In alloys (from Campbell et al. Acta Mat 2002, Development of a diffusion mobility database for Ni-base Superalloys)

$$M_{ki}^L = \delta_{ki} x_i M_i$$

$$M_i = \Theta_i \frac{1}{RT} \exp\left(\frac{\Delta Q_i^*}{RT}\right)$$

$$\Delta Q_i^* = \sum_j x_j Q_i^j + \sum_p \sum_{j > p} x_p x_j \sum_k A_i^{pj} (x_p - x_j)^k$$

## Work of Moelans & Blanpain in slags

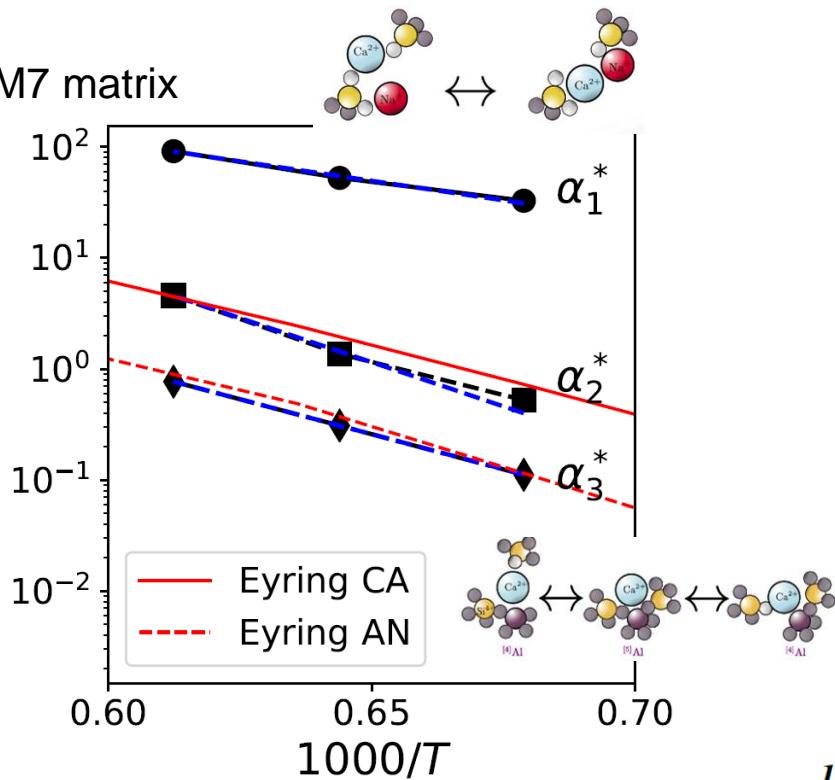
Heulens, J., Blanpain, B., & Moelans, N. (2011). A phase field model for isothermal crystallization of oxide melts. *Acta materialia*, 59(5), 2156-2165.

Liu, J., Zou, J., Guo, M., & Moelans, N. (2016). Phase field simulation study of the dissolution behavior of Al<sub>2</sub>O<sub>3</sub> into CaO-Al<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub> slags. *Computational Materials Science*, 119, 9-18.

## Calphad method

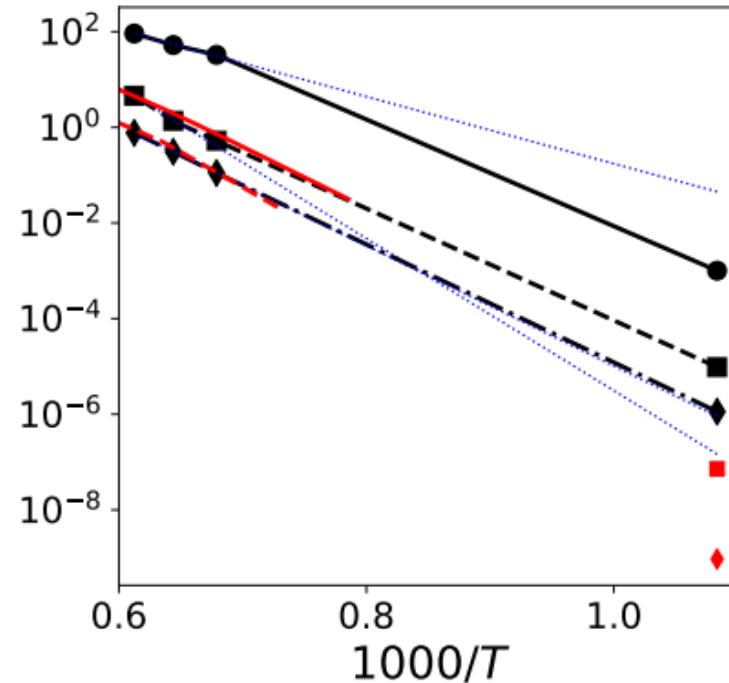
# Energetics of multicomponent diffusion

M7 matrix

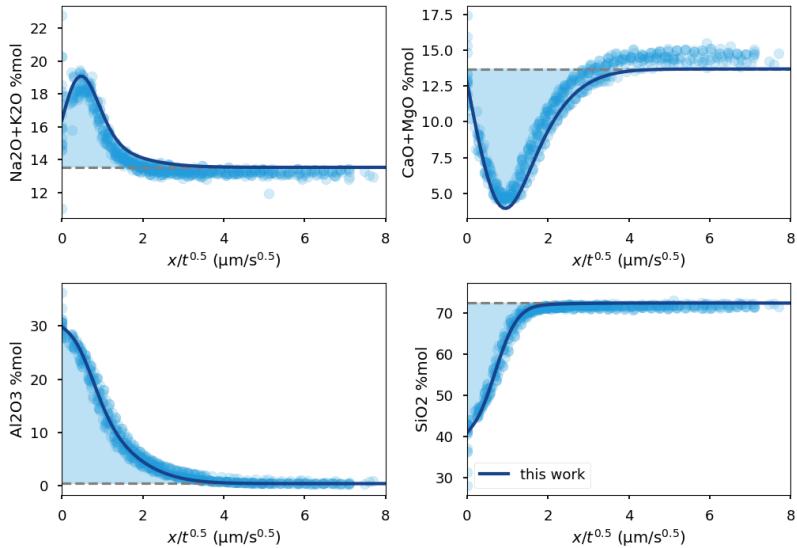
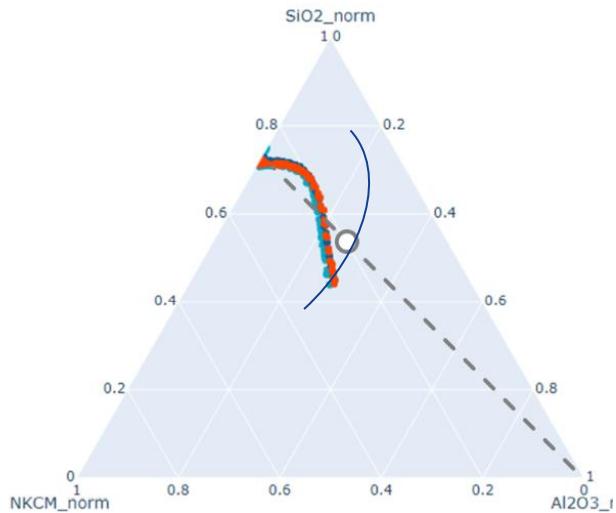
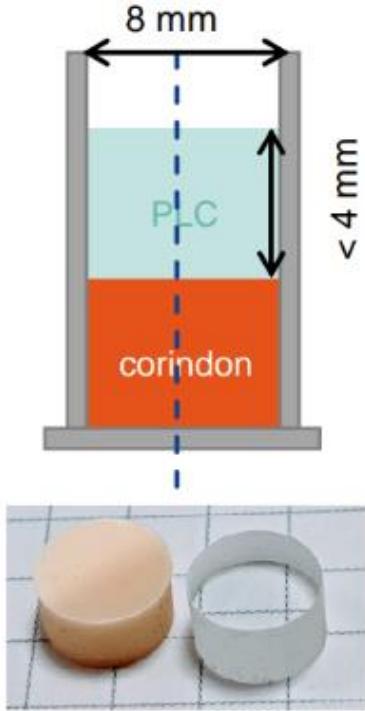


Claireaux, Corinne, et al. "Influence of temperature on multicomponent diffusion in calcium and sodium aluminosilicate melts." *Journal of Non-Crystalline Solids* 505 (2019): 170-180.

Eyring's  $\frac{k_B T}{\eta d}$   
SAINT-GC  
PARIS



# Application: dissolution of $\text{Al}_2\text{O}_3$ refractories, PhD F. Yoshizawa



Dissolution mechanisms of minerals in silicate liquids of industrial interest, F. Yoshizawa

Alumina Dissolution into Silicate Slag, Zhang et al. JACS 2000;  
Effects of melt viscosity and silica activity on the rate and mechanism of quartz dissolution in melts of the CMAS and CAS systems, Shaw 2004

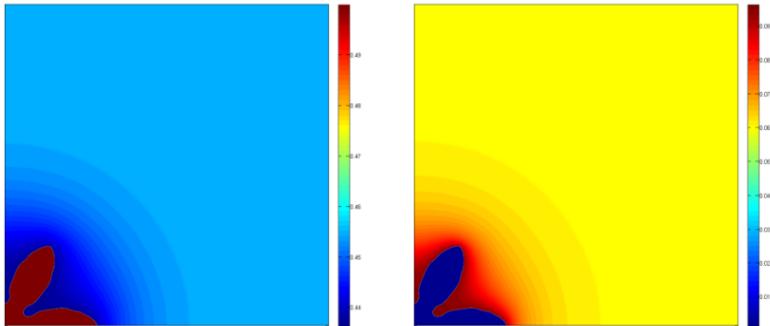
Saint-Gobain Confidential & Proprietary

SAINT-GOBAIN RESEARCH  
PARIS

Coll. Michael Toplis, Univ. Toulouse

SAINT-GOBAIN

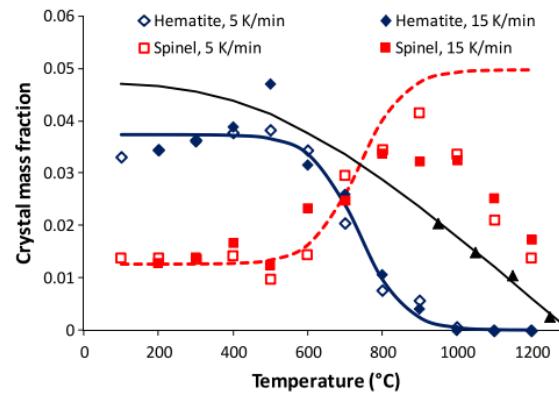
# Applications: multiphysics modelling



Heulens, J., Blanpain, B., & Moelans, N. (2011, January). Crystallization of CaO-SiO<sub>2</sub> in a CaO-Al<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub> Melt: Computer Simulations and In-situ Experiments.

How about batch dissolution rates?

Mostly phenomenological models... inspired by diffusion physics.



Pokorný, R., Rice, J. A., Crum, J. V., Schweiger, M. J., & Hrma, P. (2013). Kinetic model for quartz and spinel dissolution during melting of high-level-waste glass batch. *Journal of nuclear materials*, 443(1-3), 230-235.

Ueda, N., Vernerová, M., Kloužek, J., Ferkl, P., Hrma, P., Yano, T., & Pokorný, R. (2021). Conversion kinetics of container glass batch melting. *Journal of the American Ceramic Society*, 104(1), 34-44.

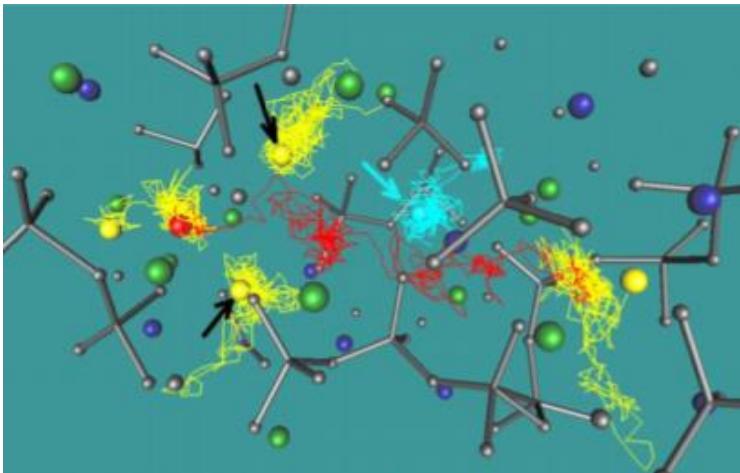
# Conclusions

Diffusion is a microscopic phenomenon, with macroscopic consequences (corrosion of refractories for example).

The complex structure of silicate melts has strong consequences about diffusion:

- $D(\text{network formers}) \ll D(\text{modifiers})$
- Strong multicomponent effects (couplings)

Diffusion is a thermally activated phenomenon.



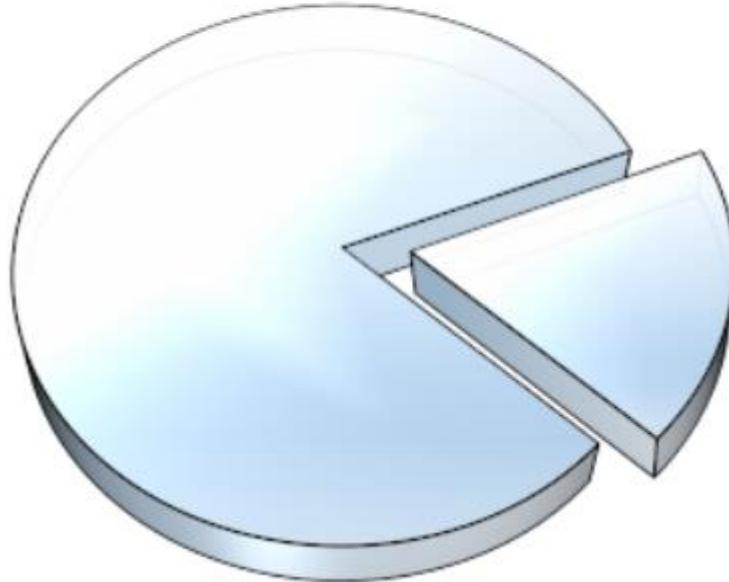
How to get & use more diffusion data?

Gather existing data from literature

Connection with thermodynamic models

Molecular dynamics and Green-Kubo formula

## GlassPy



The background of the image is a dark, smoky environment with intense orange and yellow flames on the left side. In the center-right, there are several silhouetted figures of people standing and watching the scene. The overall atmosphere is one of a dramatic or emergency event.

**THANK YOU FOR YOUR ATTENTION!**