

Multicomponent diffusion in silicate melts

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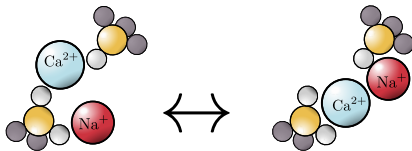
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**How to predict diffusive exchanges
in heterogeneous systems ?**

glass melting



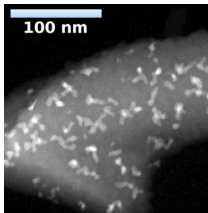
refractory corrosion



heterogeneous system
concentration gradient



chemical diffusion



Dargaud 2011

crystallization
phase separation



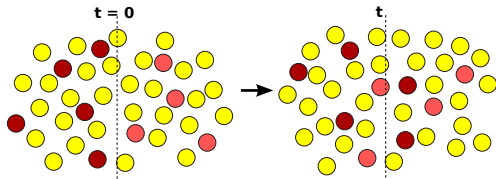
thin films on glass substrate



**How to predict diffusive exchanges
in heterogeneous systems ?**

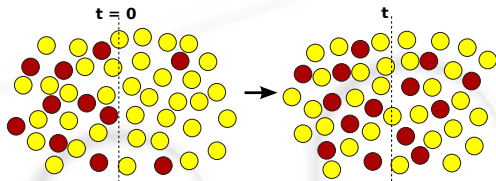
Different configurations for diffusion

Isotopic diffusion : marked tracer



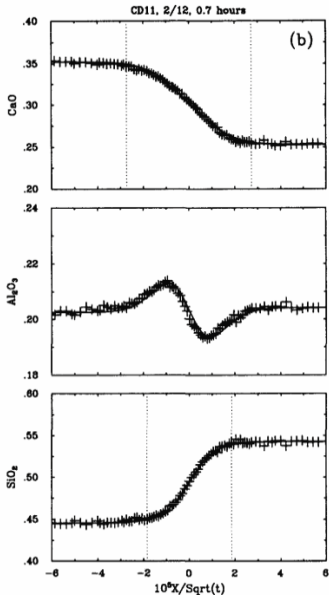
[Jambon and Carron, 1976, Richter et al., 1999]

Chemical diffusion : gradient of chemical concentration



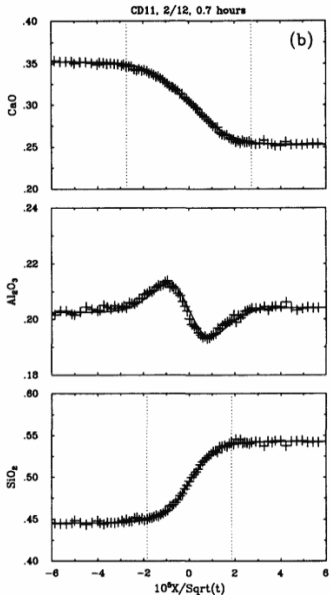
[Trial and Spera, 1994, Chakraborty et al., 1995a, Liang et al., 1996]

Interdiffusion effects : uphill diffusion

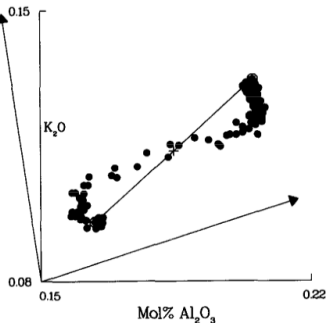


← [Liang et al., 1996]
uphill diffusion

Interdiffusion effects : uphill diffusion

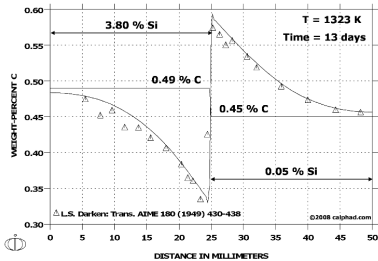


← [Liang et al., 1996]
uphill diffusion



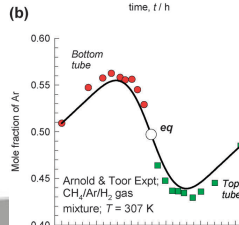
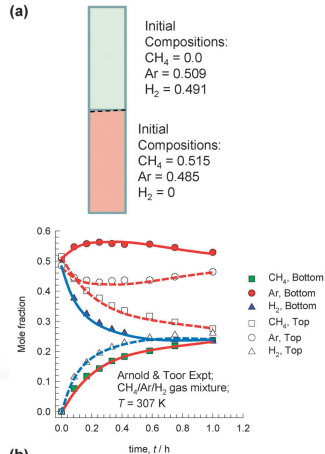
[Chakraborty et al., 1995b]
non-straight diffusion paths

Uphill diffusion in various materials



Darken, Fe-Si-C alloy

[Krishna2015]



Interdiffusion effects

Diff Couple	D(SiO ₂) (μm ² /s)	D(TiO ₂) (μm ² /s)	D(Al ₂ O ₃) (μm ² /s)	D(MgO) (μm ² /s)	D(CaO) (μm ² /s)	D(Na ₂ O) (μm ² /s)	D(K ₂ O) (μm ² /s)
Si-Ti	19.5 ± 2.8	21.5 ± 0.7					
Si-Al	15.7 ± 1.5		12.3 ± 0.8				
Si-Mg	30.0 ± 1.7			49.7 ± 1.5			
Si-Ca	28.7 ± 2.8				60.4 ± 2.0		
Si-Na	44.2 ± 4.0					401.5 ± 8.6	129.7 ± 7.4
Si-K	102.9 ± 19.5						109.1 ± 1.7
Ti-Mg		20.8 ± 0.7		46.9 ± 2.3			
Mg-Ca				61.1 ± 3.8	115.7 ± 7.2		
Ca-Na					70.4 ± 2.6	260.1 ± 3.7	
An diss		13.3 ± 0.6	17.9 ± 0.2	36.8 ± 0.5			

[Guo and Zhang, 2016]

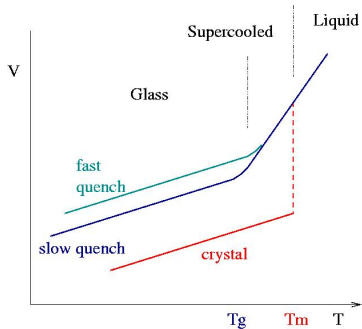
Effective binary diffusion coefficient (EBDC) depends on counter-diffusing species.

Viscous liquids quenched into amorphous solids



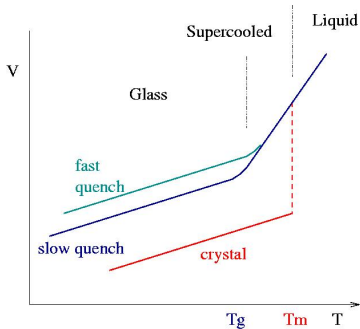
Glass transition and viscosity

Avoiding crystallization

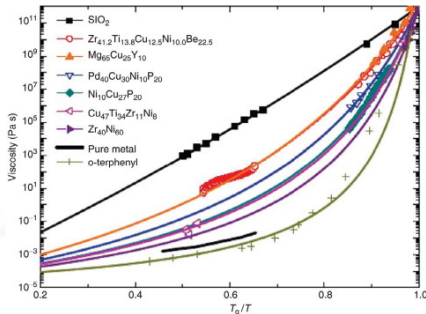


Glass transition and viscosity

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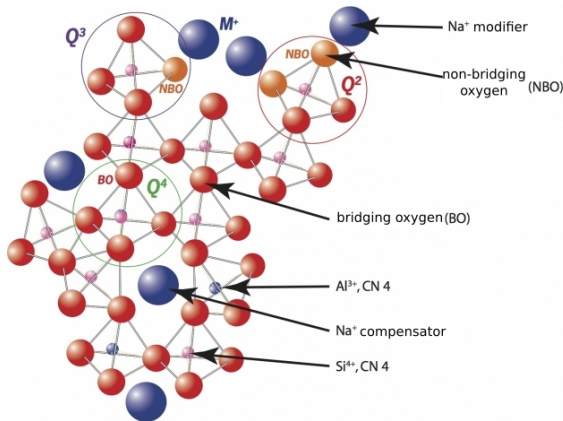


Avoiding rearrangements :
fast quenching rates or
low mobilities



Silicate glasses are strong network-forming glasses : easy to process
Downside : low mobility, high viscosity

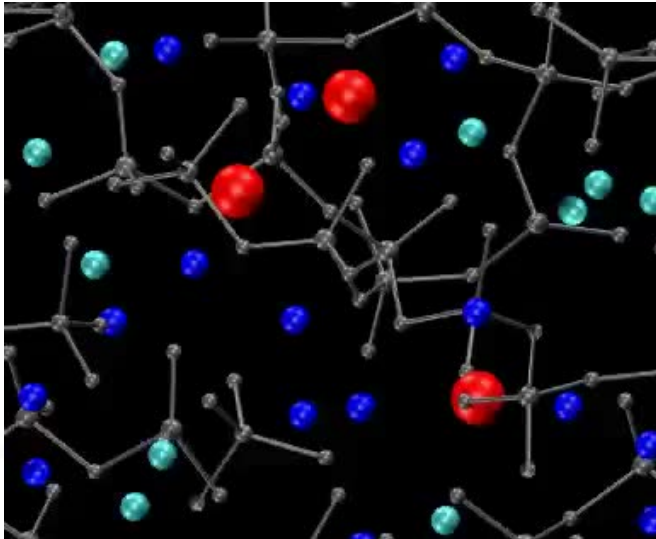
Composition and structure of silicate glasses



Polymerized network of tetrahedra

- Network formers : Si, Al, (B, P, ...)
- Network modifiers (or compensators) : Na, Ca, (K, Mg, ...)

Diffusion and reorganizations of silicate network



<https://www.youtube.com/watch?v=S0UIMspT4jw>, A. Tilocca

Diffusion matrix formalism

Fick's law

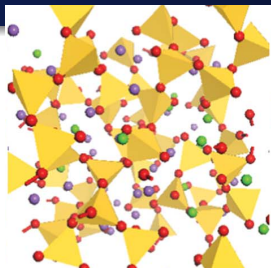
$$\mathbf{j} = -D\nabla C$$
$$\frac{\partial C}{\partial t} = D\Delta C$$



Diffusion matrix formalism

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Diffusion matrix

$$\mathbf{j} = -\mathbf{D}\nabla C$$

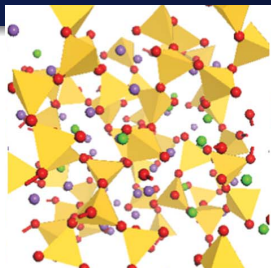
$$\mathbf{j}_i(\mathbf{x}) = -\sum_k D_{ik} \nabla C_k(\mathbf{x})$$

$$\frac{\partial}{\partial t} \begin{pmatrix} C_{\text{Na}} \\ C_{\text{Ca}} \\ C_{\text{Al}} \\ C_{\text{Si}} \end{pmatrix} = \begin{pmatrix} D_{\text{Na,Na}} & D_{\text{Na,Ca}} & D_{\text{Na,Al}} & D_{\text{Na,Si}} \\ D_{\text{Ca,Na}} & D_{\text{Ca,Ca}} & D_{\text{Ca,Al}} & D_{\text{Ca,Si}} \\ D_{\text{Al,Na}} & D_{\text{Al,Ca}} & D_{\text{Al,Al}} & D_{\text{Al,Si}} \\ D_{\text{Si,Na}} & D_{\text{Si,Ca}} & D_{\text{Si,Al}} & D_{\text{Si,Si}} \end{pmatrix} \Delta \begin{pmatrix} C_{\text{Na}} \\ C_{\text{Ca}} \\ C_{\text{Al}} \\ C_{\text{Si}} \end{pmatrix}$$

Diffusion matrix formalism

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Diffusion matrix

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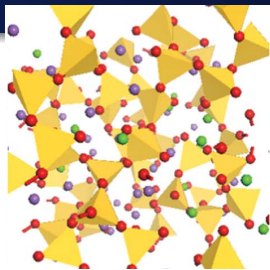
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Diffusion matrix formalism

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Measured in several ternary systems, mostly in geosciences

[Liang et al., 1996], [Richter et al., 1998] : $\text{CaO/MgO} - \text{Al}_2\text{O}_3 - \text{SiO}_2$

Also used in multicomponent metallic alloys

$$\mathbf{j}_i = -C_i M_i \nabla \mu_i = - \sum_j C_j M_j \frac{\partial \mu_i}{\partial C_j} \frac{\partial C_j}{\partial x}$$

What are the diffusion matrices in systems of industrial interest ?

- $\text{Na}_2\text{O} - \text{CaO} - \text{SiO}_2$ (NCS, W. Woelffel)
- $\text{Na}_2\text{O} - \text{Al}_2\text{O}_3 - \text{SiO}_2$ (NAS, V. Pukhkaya)
- $\text{Na}_2\text{O} - \text{CaO} - \text{Al}_2\text{O}_3 - \text{SiO}_2$ (NCAS, C. Claireaux)
- $\text{Na}_2\text{O} - \text{CaO} - \text{Al}_2\text{O}_3 - \text{SiO}_2 - \text{ZrO}_2$ (NCASZ, M. Ficheux)
- $\text{Na}_2\text{O} - \text{B}_2\text{O}_3 - \text{SiO}_2$ (NBS, H. Pablo)

How do diffusion matrices depend on composition & temperature ?

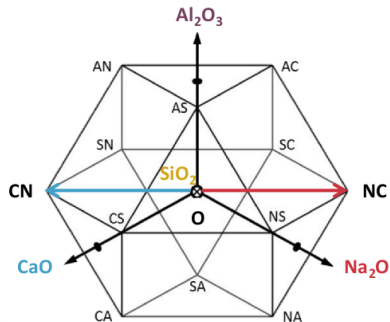
Can we predict them ?

Can we use diffusion matrices for thin films as well ?

Principle of multidiffusion experiments

[Claireaux et al., 2016] : quaternary system $\text{CaO}-\text{Na}_2\text{O}-\text{Al}_2\text{O}_3-\text{SiO}_2$

Diffusion couples centered on
64.5% SiO_2 , 13.3% Na_2O , 10.8% CaO , 11.33% Al_2O_3



12 different compositions

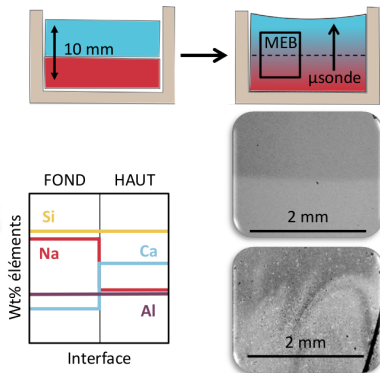
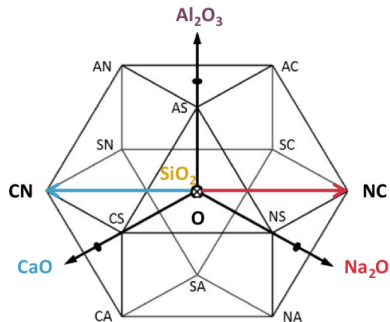
$\Delta \pm 2.5\%$

A lot of diffusion experiments!

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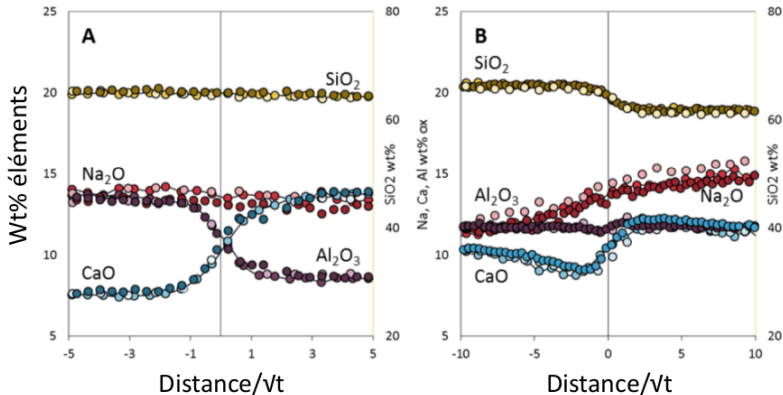
$\Delta \pm 2.5\%$

A lot of diffusion experiments !

Typical diffusion profiles

Electron microprobe profiles

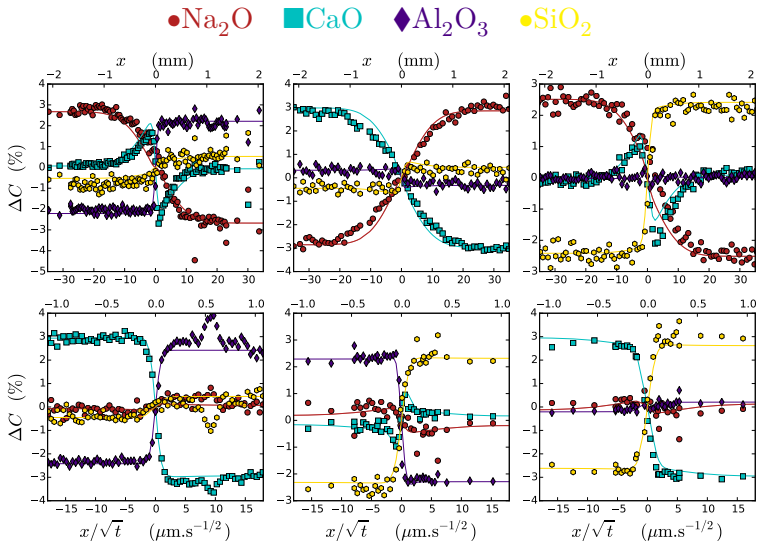
- 20 min
- 1 h
- 3 h



Good rescaling by \sqrt{t}

$$C(x, t) = f\left(\frac{x}{\sqrt{t}}\right)$$

NCAS system at 1200°C

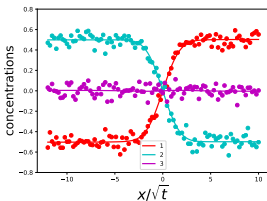
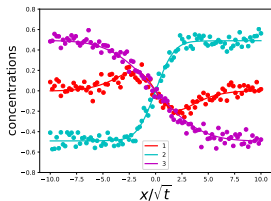


[Claireaux et al., 2016] GCA, Claireaux JNCS 2018

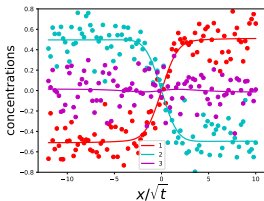
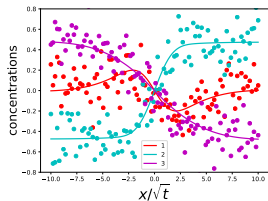
Python package to fit and simulate diffusion profiles : [multidiff](#).

Estimation of diffusion matrix

Estimating the best diffusion matrix so that generated theoretical diffusion profiles fit optimally experimental diffusion profiles.



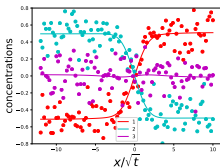
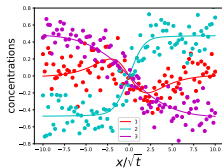
low noise



more noise

Estimation of diffusion matrix

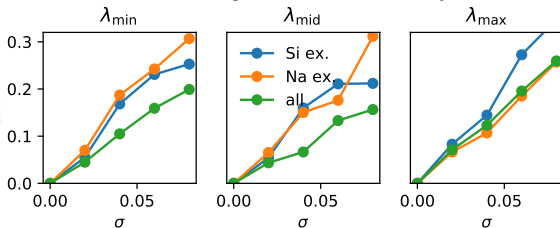
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more noise

The more experiments (and data points), the better !

Relative error on eigenvalues vs. intensity of noise



Python package to fit and simulate diffusion profiles : [multidiff](#).

Diffusion matrix at 1200° C

	GRADIENTS			
	∇_{Na}	∇_{Ca}	∇_{Al}	∇_{Si}
J_{Na}	19	-13	-14	-10
J_{Ca}	-16	12	13	8.9
J_{Al}	-0.23	-0.17	0.08	0.03
J_{Si}	-2	0.75	0.97	1

$\mu\text{m}^2/\text{s}$

- Not symmetric
- Large off-diagonal terms

The matrix can be used to compute the evolution of **any** diffusion couple (in the domain of interest).

Diffusion matrix formalism

Let us define

$$\mathbf{D} = \mathbf{P}\mathbf{L}\mathbf{P}^{-1},$$

with

$$\mathbf{L} = \begin{pmatrix} \lambda_1 & \dots & \dots \\ \vdots & \ddots & \dots \\ \vdots & \vdots & \lambda_{n-1} \end{pmatrix}$$

the diagonal matrix of **eigenvalues** λ_i , and $\mathbf{P} = (\mathbf{v}_1, \dots, \mathbf{v}_{n-1})$ is the block matrix of the **eigenvectors** \mathbf{v}_i : $\mathbf{L}\mathbf{v}_i = \lambda_i\mathbf{v}_i$

With $\tilde{\mathbf{C}} = \mathbf{P}^{-1}\mathbf{C}$ and $\tilde{\Delta}\mathbf{C} = \mathbf{P}^{-1}\Delta\mathbf{C}$,

$$\frac{\partial \tilde{\mathbf{C}}}{\partial t} = \mathbf{L}\nabla^2 \tilde{\mathbf{C}}$$

$$\tilde{C}_i(x, t) = \Delta\tilde{C}_i \operatorname{erf}\left(\frac{x}{\sqrt{2\lambda_i t}}\right).$$

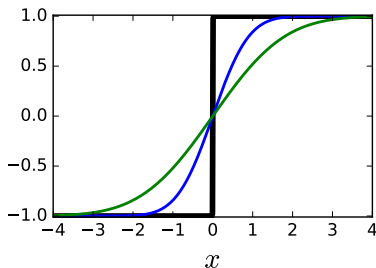
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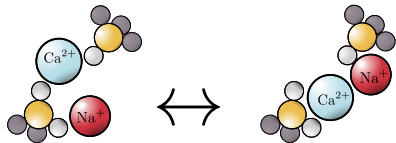
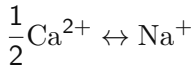
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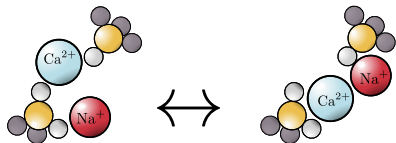
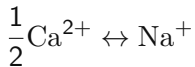
Diffusion eigenvectors at 1200° C

Dominant eigenvector, $\lambda_1 \simeq 30 \cdot 10^{-12} \text{m}^2 \cdot \text{s}^{-1}$

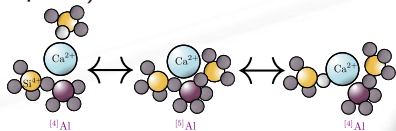


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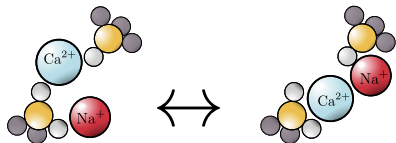
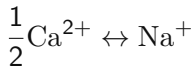


Second eigenvector (52x less frequent)

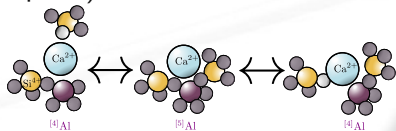


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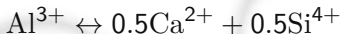
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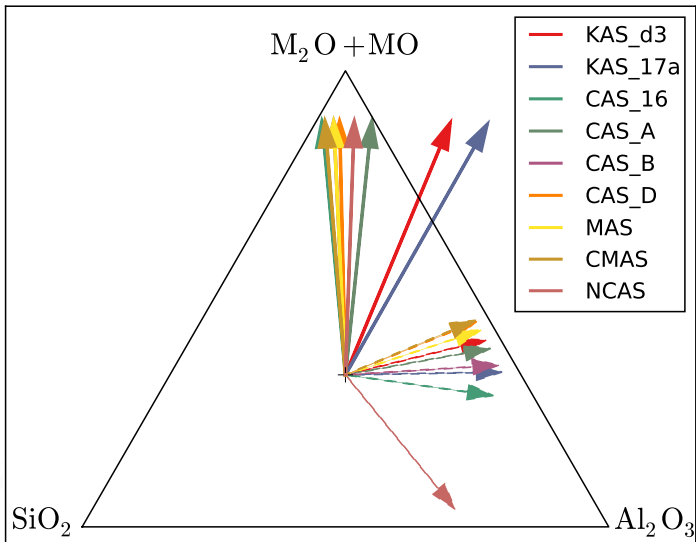
Second eigenvector (52x less frequent)



Third eigenvector (155x less frequent)

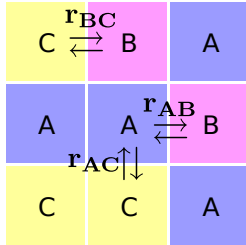


composition	eigenvectors	eigenvalues $\times 10^{-12} \text{ m}^2 \cdot \text{s}^{-1}$	T (° C)	ref.
NCS 16-12-72	$\text{Na}_2\text{O} \leftrightarrow 0.85\text{CaO} + 0.15\text{SiO}_2$ $0.27\text{Na}_2\text{O} + 0.73\text{CaO} \leftrightarrow \text{SiO}_2$	105 4.4	1200	(Trial and Spera [114])
KSrS 21-17-62	$0.86\text{K}_2\text{O} + 0.14\text{SiO}_2 \leftrightarrow \text{SrO}$ $\text{SrO} \leftrightarrow \text{SiO}_2$	0.09 0.002	806	(Varshneya and Cooper [118])
KAS D-3 9-16-75	$\text{K}_2\text{O} \leftrightarrow 0.14\text{Al}_2\text{O}_3 + 0.86\text{SiO}_2$ $0.75\text{Al}_2\text{O}_3 + 0.25\text{K}_2\text{O} \leftrightarrow \text{SiO}_2$	0.07 1×10^{-3}	1400	(Chakraborty et al. [19])
KAS 17a 16-9-75	$\text{K}_2\text{O} \leftrightarrow 0.01\text{Al}_2\text{O}_3 + 0.99\text{SiO}_2$ $0.98\text{Al}_2\text{O}_3 + 0.02\text{K}_2\text{O} \leftrightarrow \text{SiO}_2$	10.8 0.04	1400	(Chakraborty et al. [19])
KAS 23 8-17-74	$\text{K}_2\text{O} \leftrightarrow 0.02\text{Al}_2\text{O}_3 + 0.98\text{SiO}_2$ $0.8\text{Al}_2\text{O}_3 + 0.02\text{K}_2\text{O} \leftrightarrow \text{SiO}_2$	14.4 0.08	1600	(Chakraborty et al. [19])
NKASH	$\text{Na}_2\text{O} \leftrightarrow \text{SiO}_2$ $\text{K}_2\text{O} \leftrightarrow \text{SiO}_2$ $\text{Al}_2\text{O}_3 + 0.21\text{Na}_2\text{O} + 0.28\text{H}_2\text{O} \leftrightarrow 1.57\text{SiO}_2$ $\text{H}_2\text{O} \leftrightarrow \text{SiO}_2$	550 540 3.4 280	1600	(Mungall et al. [78])
CAS 16 30-20-50	$\text{CaO} \leftrightarrow 0.58\text{Al}_2\text{O}_3 + 0.42\text{SiO}_2$ $\text{Al}_2\text{O}_3 \leftrightarrow 0.15\text{CaO} + 0.85\text{SiO}_2$	99 23	1500	(Liang et al. [67])
CAS A 25-13-62	$\text{CaO} \leftrightarrow 0.41\text{Al}_2\text{O}_3 + 0.59\text{SiO}_2$ $0.81\text{Al}_2\text{O}_3 + 0.19\text{CaO} \leftrightarrow \text{SiO}_2$	34 8.3	1500	(Liang et al. [67])
CAS B 28-15-57	$\text{CaO} \leftrightarrow 0.54\text{Al}_2\text{O}_3 + 0.46\text{SiO}_2$ $0.93\text{Al}_2\text{O}_3 + 0.07\text{CaO} \leftrightarrow \text{SiO}_2$	47 23	1500	(Liang et al. [67])
CAS D 23-15-62	$\text{CaO} \leftrightarrow 0.52\text{Al}_2\text{O}_3 + 0.48\text{SiO}_2$ $0.62\text{Al}_2\text{O}_3 + 0.38\text{CaO} \leftrightarrow \text{SiO}_2$	36 11	1500	(Liang et al. [67])
MAS 22-18-60	$\text{MgO} \leftrightarrow 0.54\text{Al}_2\text{O}_3 + 0.46\text{SiO}_2$ $0.68\text{Al}_2\text{O}_3 + 0.32\text{MgO} \leftrightarrow \text{SiO}_2$	70 20	1550	(Richter et al. [90])
CMAS 9-7-20-64	$\text{CaO} \leftrightarrow 0.36\text{MgO} + 0.32\text{Al}_2\text{O}_3 + 0.32\text{SiO}_2$ $\text{MgO} \leftrightarrow 0.07\text{CaO} + 0.53\text{Al}_2\text{O}_3 + 0.4\text{SiO}_2$ $0.13\text{CaO} + 0.26\text{MgO} + 0.61\text{Al}_2\text{O}_3 \leftrightarrow \text{SiO}_2$	59 26 3.2	1500	(Richter et al. [90])
NCAS 13-11-11-65	$\text{Na}_2\text{O} + 0.02\text{Al}_2\text{O}_3 \leftrightarrow 0.92\text{CaO} + 0.09\text{SiO}_2$ $\text{CaO} + 0.02\text{Na}_2\text{O} \leftrightarrow 0.47\text{Al}_2\text{O}_3 + 0.52\text{SiO}_2$ $\text{CaO} + 0.17\text{SiO}_2 \leftrightarrow 0.99\text{Al}_2\text{O}_3 + 0.15\text{Na}_2\text{O}$	29.5 0.58 0.3	1200	(Chaireux et al. [24])
NAS 19-9-72	$\text{Na}_2\text{O} \leftrightarrow \text{SiO}_2$ $\text{SiO}_2 \leftrightarrow \text{Al}_2\text{O}_3$	0.74 0.04	1200	
BNS 18-14-68	$\text{Na}_2\text{O} \leftrightarrow 0.9\text{SiO}_2 + 0.1\text{B}_2\text{O}_3$ $\text{SiO}_2 \leftrightarrow 0.75\text{B}_2\text{O}_3 + 0.25\text{Na}_2\text{O}$	10.1 0.5	1100	(Pablo et al. [82])



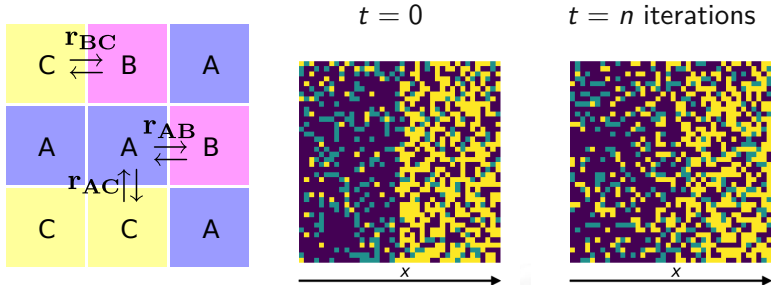
A toy model for multicomponent diffusion

Random exchange of neighbors with fixed probability r_{AB}



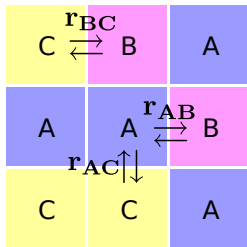
A toy model for multicomponent diffusion

Random exchange of neighbors with fixed probability r_{AB}

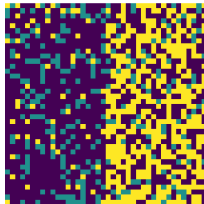


A toy model for multicomponent diffusion

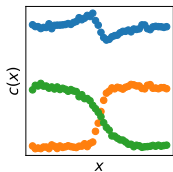
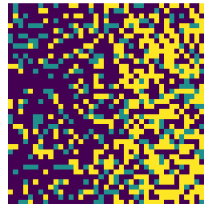
Random exchange of neighbors with fixed probability r_{AB}



$t = 0$

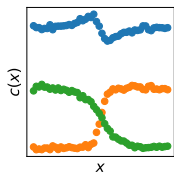
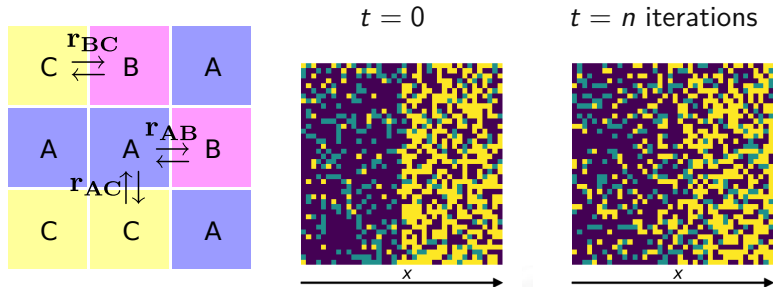


$t = n$ iterations



A toy model for multicomponent diffusion

Random exchange of neighbors with fixed probability r_{AB}



Compute diffusion matrix from exchange rates r_{ij}

$$\mathbf{D} = \frac{1}{3} \begin{pmatrix} (1 - c_2)r_{13} + c_2r_{12} & c_1(r_{13} - r_{12}) \\ c_2(r_{23} - r_{12}) & (1 - c_1)r_{23} + c_1r_{12} \end{pmatrix}$$

A different view on microscopic dynamics

composition	eigenvectors	eigenvalues $\times 10^{-12} \text{m}^2 \cdot \text{s}^{-1}$	T (° C)
NCS 16-12-72	$\text{Na}_2\text{O} \leftrightarrow 0.85\text{CaO} + 0.15\text{SiO}_2$ $0.27\text{Na}_2\text{O} + 0.73\text{CaO} \leftrightarrow \text{SiO}_2$	105 4.4	1200
NCAS 13-11-11-65	$\text{Na}_2\text{O} + 0.02\text{Al}_2\text{O}_3 \leftrightarrow 0.92\text{CaO} + 0.09\text{SiO}_2$ $\text{CaO} + 0.02\text{Na}_2\text{O} \leftrightarrow 0.47\text{Al}_2\text{O}_3 + 0.52\text{SiO}_2$ $\text{CaO} + 0.17\text{SiO}_2 \leftrightarrow 0.99\text{Al}_2\text{O}_3 + 0.15\text{Na}_2\text{O}$	29.5 0.58 0.3	1200
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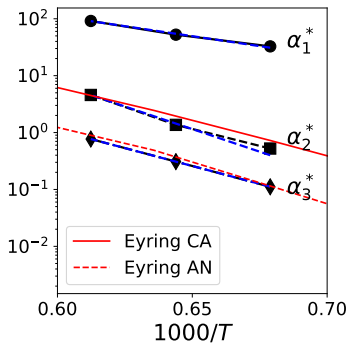
composition	exchange rates		
NCS	$r_{NC} = 3.2$	$r_{NS} = 1.3$	$r_{CS} = 0$
NCAS	$r_{NC} = 1$ $r_{CA} = 0$	$r_{NA} = 0.4$ $r_{CS} = 0$	$r_{NS} = 0.4$ $r_{AS} = 0$
BNS	$r_{BN} = 0.2$	$r_{BS} = 0$	$r_{NS} = 0.2$

Stoichiometry of diffusion eigenvectors related to

- binary exchange rates
- ... but also oxide concentrations

Energetics of diffusion matrices

NCAS melts, temperature dependence of eigenvalues

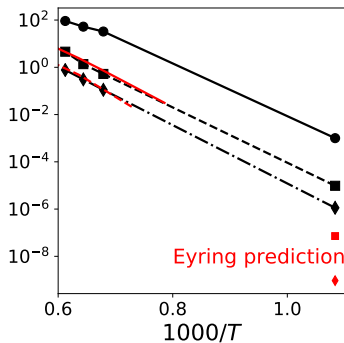
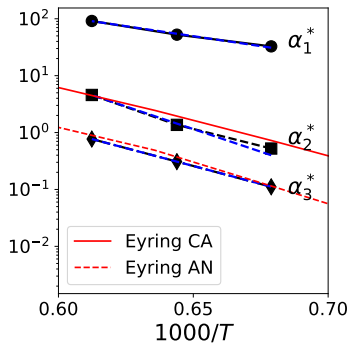


High-temperature :

- Arrhenian behavior
- Eyring relation OK

Energetics of diffusion matrices

NCAS melts, temperature dependence of eigenvalues



High-temperature :

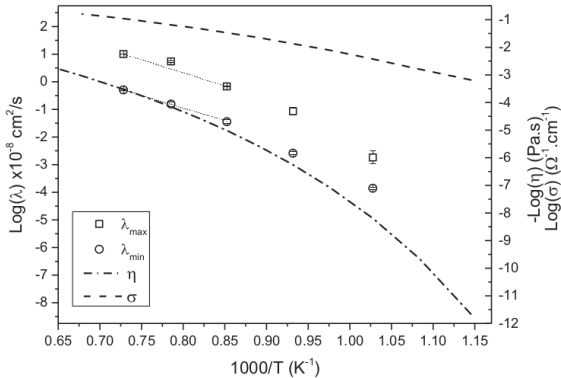
- Arrhenian behavior
- Eyring relation OK

Close to T_g

- Still Arrhenian
- Breakdown of Eyring relation

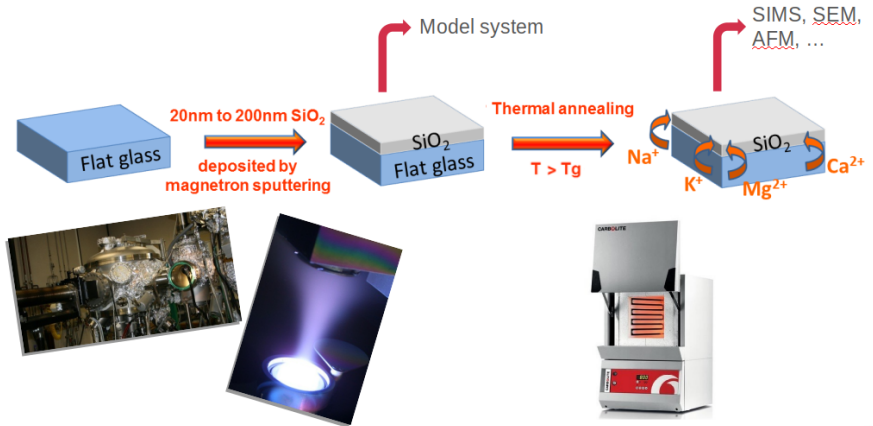
Energetics of diffusion matrices

Same behavior observed in sodium borosilicate composition [Pablo et al., 2017]



Arrhenian + breakdown of Eyring relation close to T_g

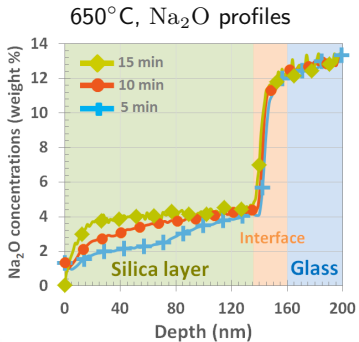
Annealing of PVD-sputtered silica layers on soda-lime substrate (Planiclear)



Pure and Al-doped silica thin films

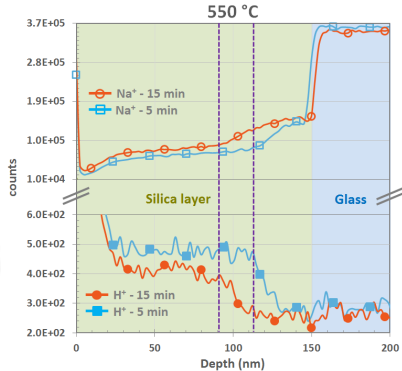
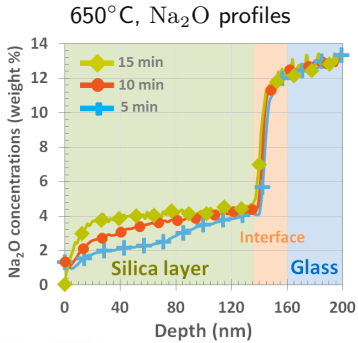
Uptake of alkali ions in silica layers

Exchange between protons and alkali ions



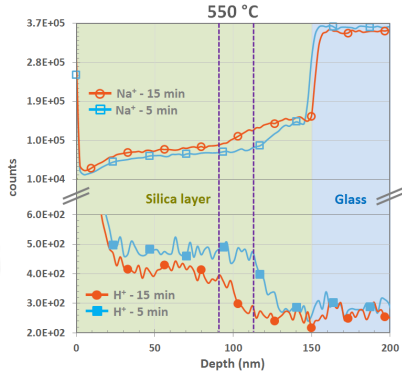
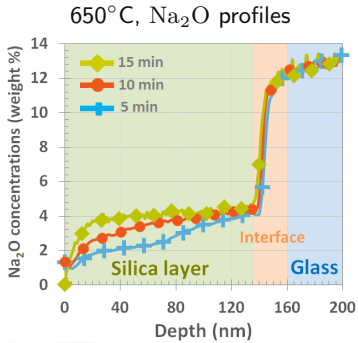
Uptake of alkali ions in silica layers

Exchange between protons and alkali ions

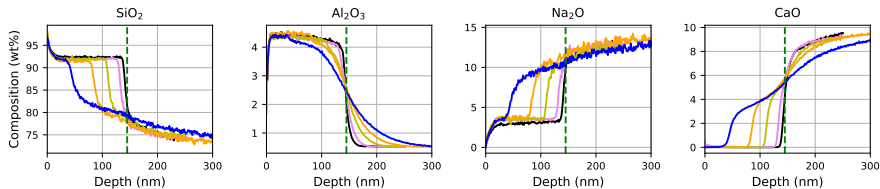


Uptake of alkali ions in silica layers

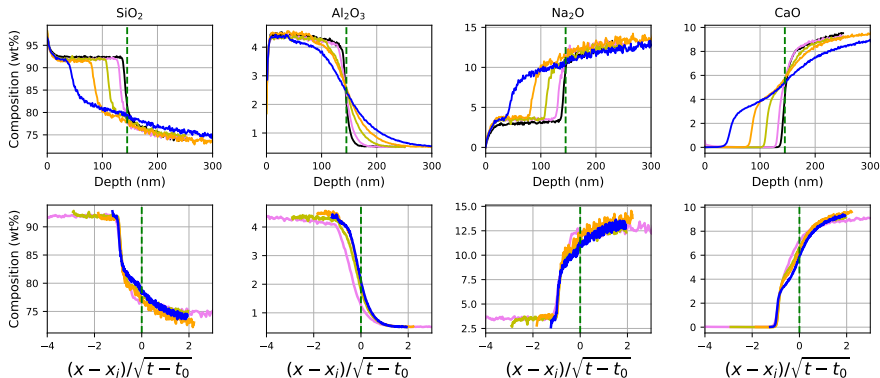
Exchange between protons and alkali ions



Diffusive dissolution of thin film and multicomponent effects

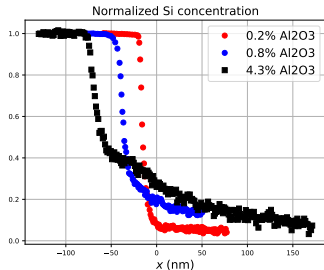


Diffusive dissolution of thin film and multicomponent effects



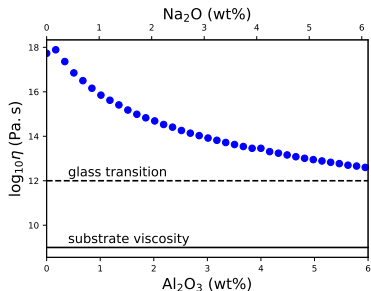
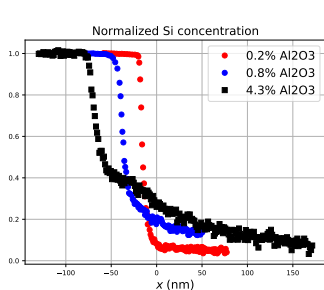
- Diffusion distance of Al smaller than for Si
- Na coupled to Si, Ca to both Si and Al.
- Can we use the bulk diffusion matrix to explain these results ?

Fitting asymmetric diffusion profiles



High Si diffusivity (& viscosity) ratio between substrate and film

Fitting asymmetric diffusion profiles

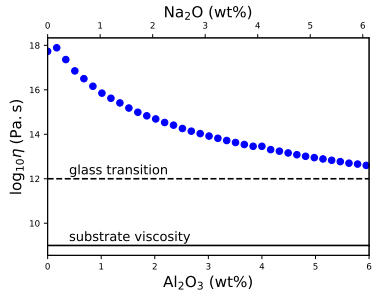
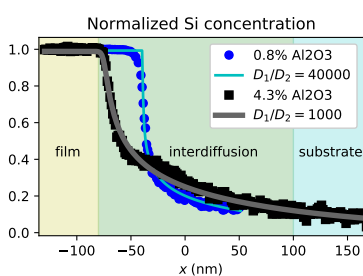


High Si diffusivity (& viscosity) ratio between substrate and film
Using Crank's model to fit profiles :

$$D_{\text{Si}} = D_0 \exp(-\beta C_{\text{Si}})$$

Fitted values of β consistent with Eyring's law and viscosity model
(Priven)

Fitting asymmetric diffusion profiles

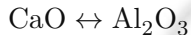
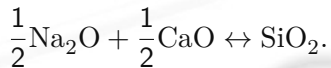
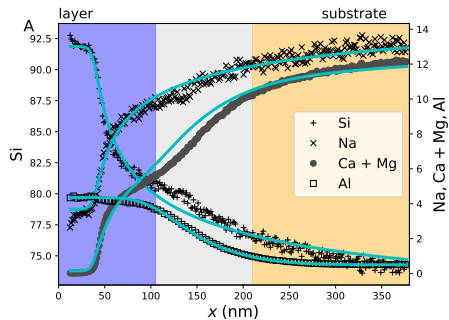


High Si diffusivity (& viscosity) ratio between substrate and film
Using Crank's model to fit profiles :

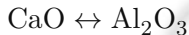
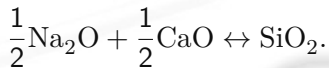
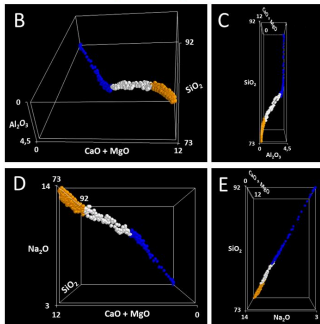
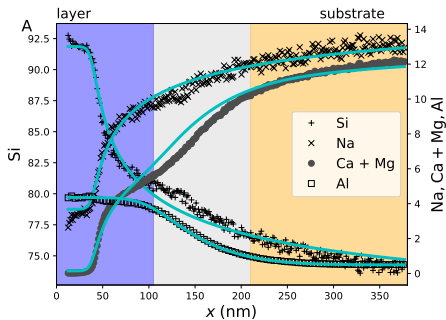
$$D_{Si} = D_0 \exp(-\beta C_{Si})$$

Fitted values of β consistent with Eyring's law and viscosity model
(Priven)

Multicomponent diffusion consistent between bulk and thin films



Multicomponent diffusion consistent between bulk and thin films



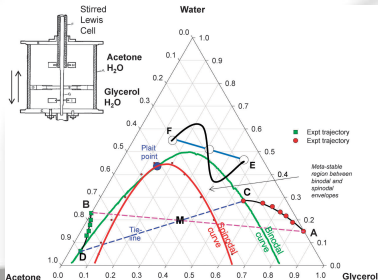
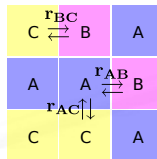
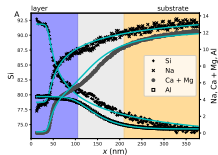
Conclusions

Diffusion matrices : a powerful tool to predict diffusive exchanges (useful outside of geochemistry !)

Diffusion matrix coefficients are related to concentration and local mobility (exchange rates).

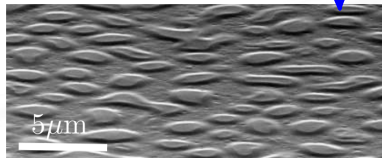
Link with structure ?

Link with phase separation, crystallization, oxidation-reduction, ... ?



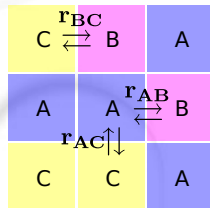
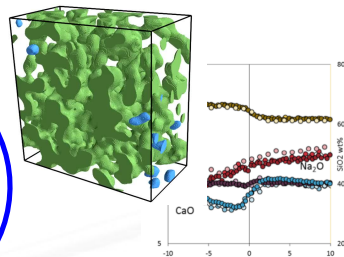
Conclusions

Industrial systems & questions



Materials properties and design

Controlled experiments



Physics models

Bibliography I



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Multicomponent diffusion in ternary silicate melts in the system $K_2O-Al_2O_3-SiO_2$: I. experimental measurements.

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Chakraborty, S., Dingwell, D. B., and Rubie, D. C. (1995b).

Multicomponent diffusion in ternary silicate melts in the system $K_2O-Al_2O_3-SiO_2$: II. mechanisms, systematics, and geological applications.

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Richter, F. M., Liang, Y., and Minarik, W. G. (1998).
Multicomponent diffusion and convection in molten $\text{MgO}-\text{Al}_2\text{O}_3-\text{SiO}_2$.
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Geochimica et Cosmochimica Acta, 58(18) :3769–3783.

Conclusions

- Diffusion matrices : a powerful tool (useful outside of geochemistry !)
- Multicomponent effects modeled on bulk and thin films
- Contrast of transport properties have to be modeled
- Exchanges with atmosphere cannot be neglected for thin films, role of water and Al content

