



How to assess the structure of glasses ?

CNRS thematic school about glass structure



Structure of disordered materials by neutron diffraction

Gabriel Cuello
Institut Laue Langevin
Grenoble, France

cuello@ill.eu

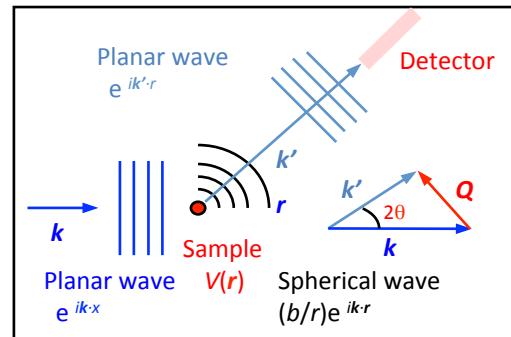
Characterization of glass structure
18 - 22 November 2019
EPN Campus – Grenoble, France



Scattering experiment

Experiment

Microscopic properties



Intensity
 $I(2\theta, \omega)$

Scattering Cross Section
 $d^2\sigma/d\Omega d\omega$

Dynamical SF
 $S(Q, \omega)$

$$I(2\theta, \omega) = C \Phi_0 \frac{d^2\sigma}{d\sigma d\omega}(2\theta, \omega) \epsilon(k')$$

Beam

Sample

Detector

$$\frac{d^2\sigma}{d\sigma d\omega}(2\theta, \omega) = N \frac{k'}{k} \frac{\sigma}{4\pi} S(\vec{Q}, \omega)$$

Interaction System

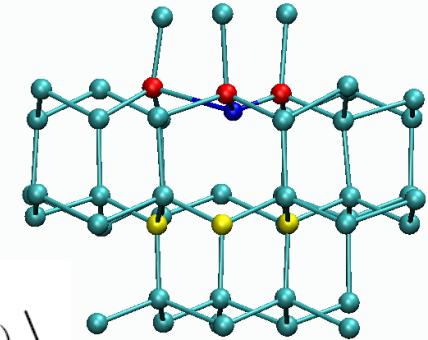


Microscopic properties

Dynamical Structure Factor $S(\vec{Q}, \omega)$

Microscopic Configuration

$$\{\mathbf{r}_1(t), \mathbf{r}_2(t), \dots, \mathbf{r}_N(t)\}$$



$$S(\vec{Q}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{-i\omega t} \frac{1}{N} \sum_{i,j}^N \left\langle e^{-i\vec{Q} \cdot \vec{r}_i(0)} e^{-i\vec{Q} \cdot \vec{r}_j(t)} \right\rangle$$

$$\delta(\vec{r}) = \frac{1}{(2\pi)^3} \int_{\text{all } \vec{Q}} e^{i\vec{Q} \cdot \vec{r}} d\vec{Q} \quad \rho(\vec{r}, t) = \sum_{i=1}^N \delta(\vec{r} - \vec{r}_i(t))$$

Microscopic particle density

$$S(\vec{Q}, \omega) = \frac{1}{2\pi} \iint d\vec{r} dt e^{i(\vec{Q} \cdot \vec{r} - \omega t)} G(\vec{r}, t)$$

Probability density of having a given atom somewhere,
e.g. at $(0, 0)$, and any atom at (\mathbf{r}, t)

van Hove's correlation function

$$G(\vec{r}, t) = \frac{1}{N} \int d\vec{r}' \left\langle \rho(\vec{r}', 0) \rho(\vec{r} + \vec{r}', t) \right\rangle$$



Coherent and incoherent scattering

$$\frac{d^2\sigma}{d\Omega d\omega}(2\theta, \omega) = N \frac{k'}{k} \left\{ \frac{\sigma_{coh}}{4\pi} S(\vec{Q}, \omega) + \left(\frac{\sigma_{inc}}{4\pi} \right) S_s(\vec{Q}, \omega) \right\}$$

coherent

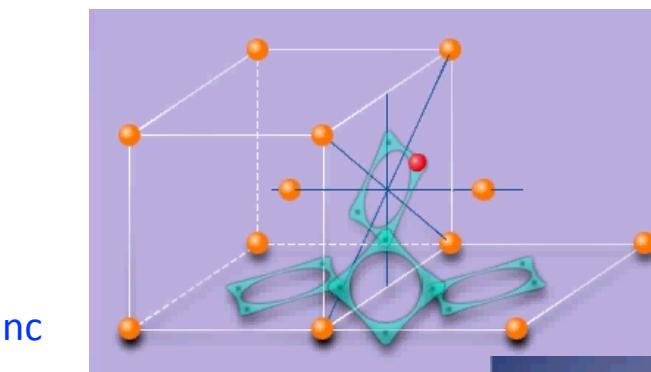
incoherent

$$\sigma_{coh} = 4\pi |\langle b \rangle|^2 \quad \text{squared mean}$$

$$\sigma_{inc} = 4\pi \left(\langle |b|^2 \rangle - |\langle b \rangle|^2 \right) \text{variance}$$

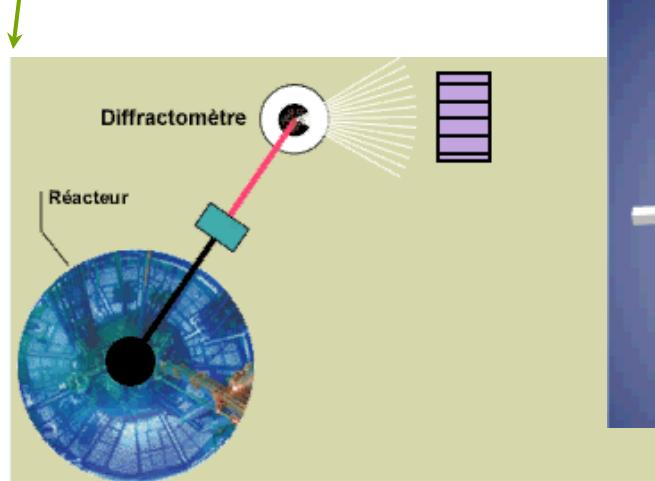
Scattering cross section

$$\Sigma = \sigma_{coh} + \sigma_{inc}$$



Structural information
Collective dynamics

Diffraction
(no energy discrimination)



Diffusive Motions
Internal Dynamics



Static structure factor

$$S(\vec{Q}) = \int_{-\infty}^{+\infty} d\omega S(\vec{Q}, \omega) = \int d\vec{r} e^{i\vec{Q}\cdot\vec{r}} G(\vec{r}, 0)$$

Static approximation

$S(Q)-1$ &
 $g(r)-1$
become a
FT pair

$$S(\vec{Q}) - 1 = \rho \int_V d\vec{r} [g(\vec{r}) - 1] e^{i\vec{Q}\cdot\vec{r}}$$

$$\rho [g(\vec{r}) - 1] = \frac{1}{(2\pi)^3} \int d\vec{Q} [S(\vec{Q}) - 1] e^{-i\vec{Q}\cdot\vec{r}}$$

Definition

$$F(\vec{Q}) = S(\vec{Q}) - 1$$

$$G(\vec{r}) = 4\pi\rho r [g(\vec{r}) - 1]$$

$$F(\vec{Q}) = \int_V d\vec{r} \frac{G(\vec{r})}{4\pi r} e^{i\vec{Q}\cdot\vec{r}}$$

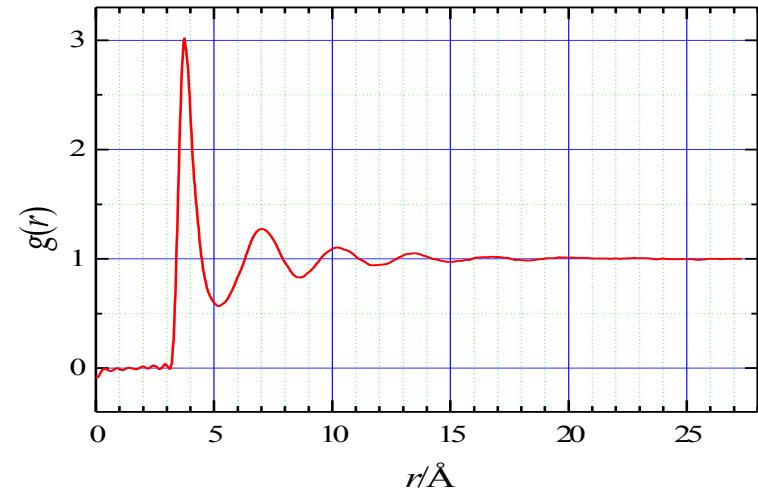
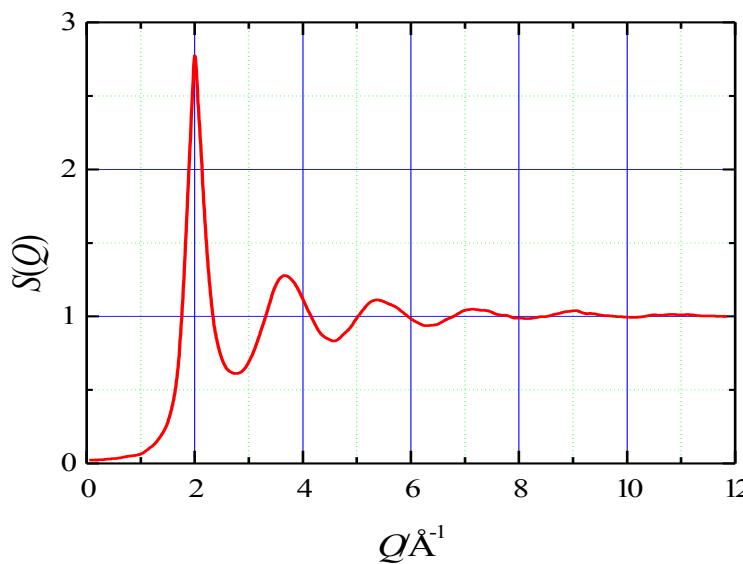
$$\frac{G(\vec{r})}{4\pi r} = \frac{1}{(2\pi)^3} \int d\vec{Q} F(\vec{Q}) e^{-i\vec{Q}\cdot\vec{r}}$$



Isotropic case

$$Q F(Q) = \int_0^\infty G(r) \sin(Qr) dr$$

$$G(r) = \frac{2}{\pi} \int_0^\infty Q F(Q) \sin(Qr) dQ$$



$$S(Q) - 1 = \frac{4\pi\rho}{Q} \int_0^\infty r [g(r) - 1] \sin(Qr) dr$$

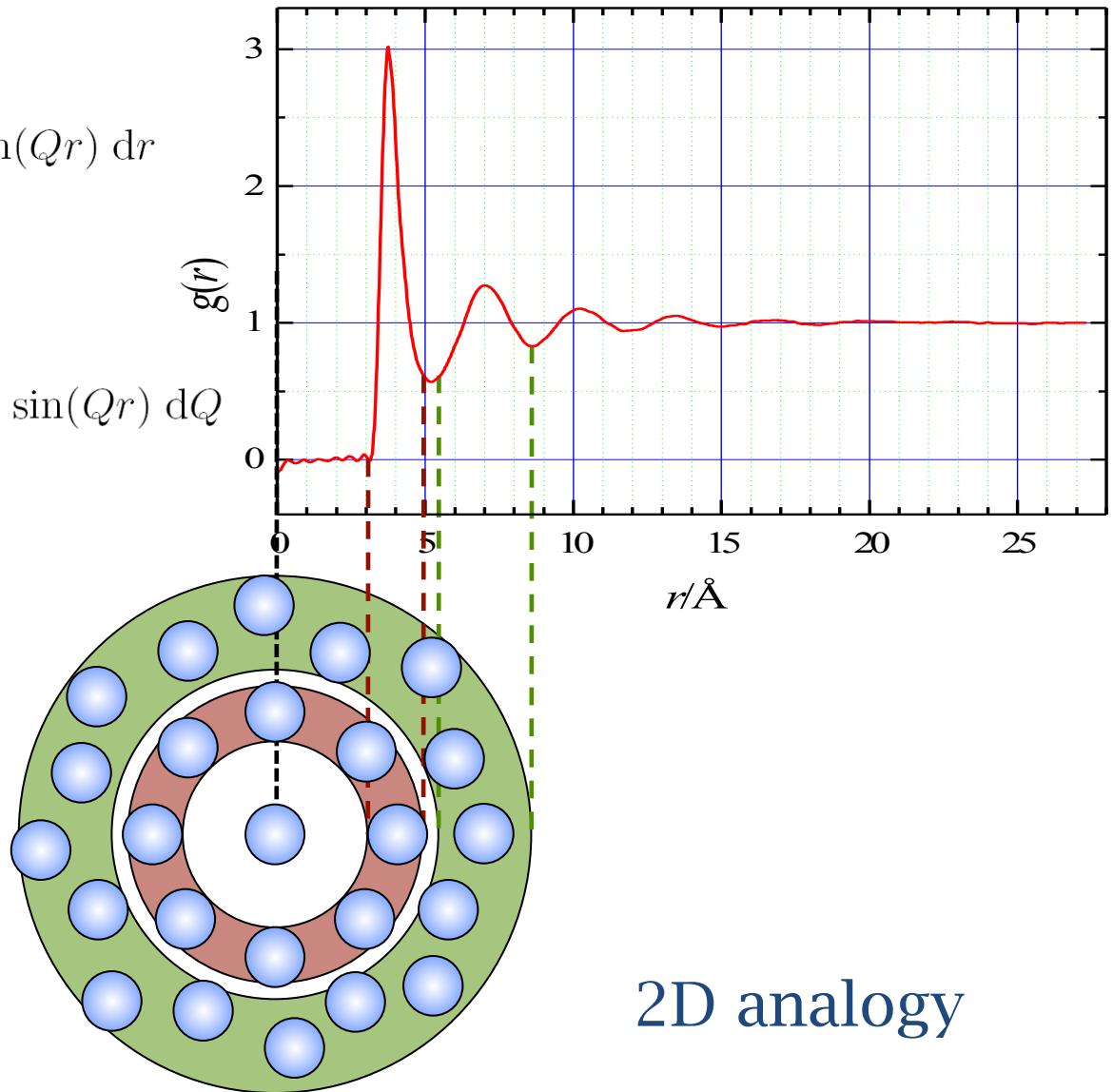
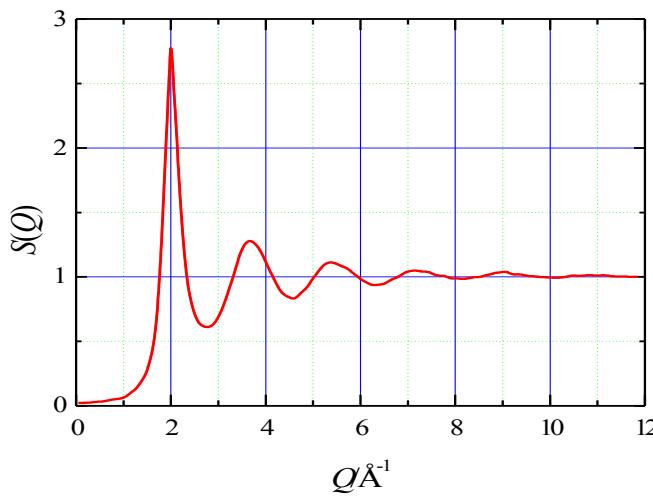
$$g(r) - 1 = \frac{1}{2\pi^2\rho r} \int_0^\infty Q [S(Q) - 1] \sin(Qr) dQ$$

Pair distribution function



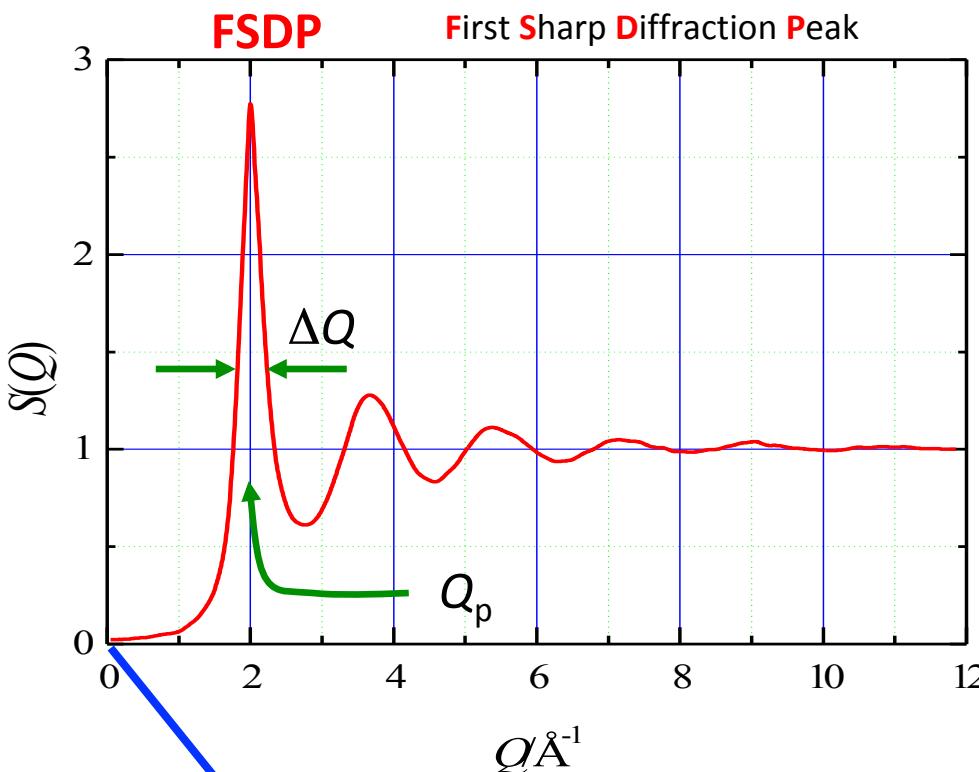
$$S(Q) - 1 = \frac{4\pi\rho}{Q} \int_0^\infty r [g(r) - 1] \sin(Qr) dr$$

$$g(r) - 1 = \frac{1}{2\pi^2\rho r} \int_0^\infty Q [S(Q) - 1] \sin(Qr) dQ$$





From $S(Q)$ to $g(r)$



$$S(0) = \rho \chi_T k_B T$$

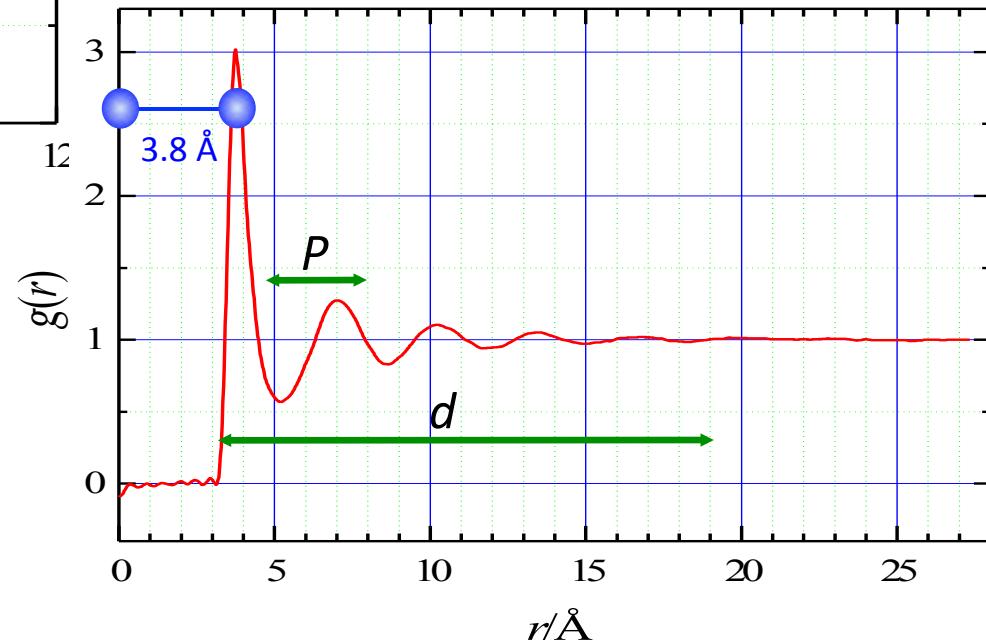
Limiting values → Normalisation

Liquid Ar @ 85K

J.L. Yarnell *et al.* (1973) PRA 7, 2130

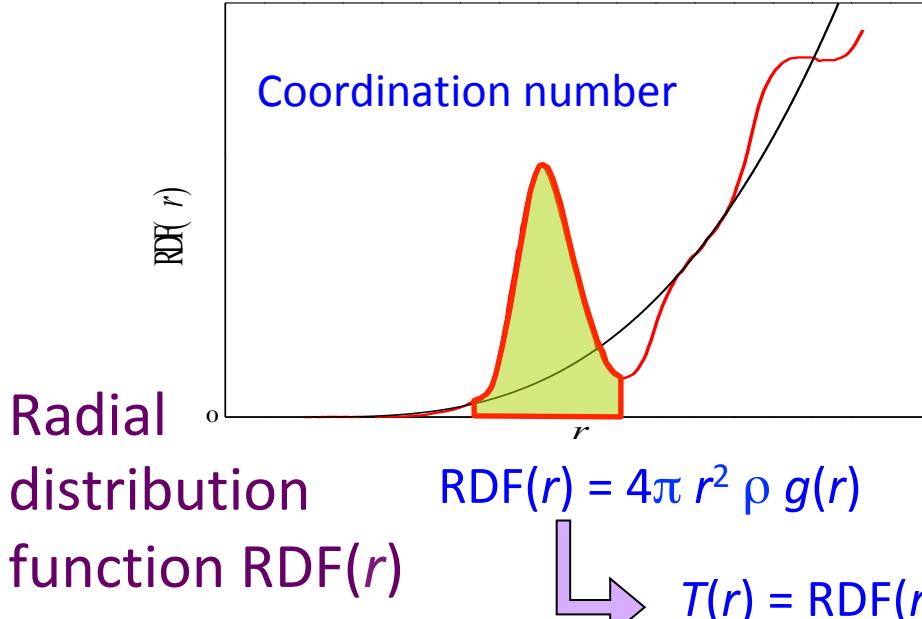
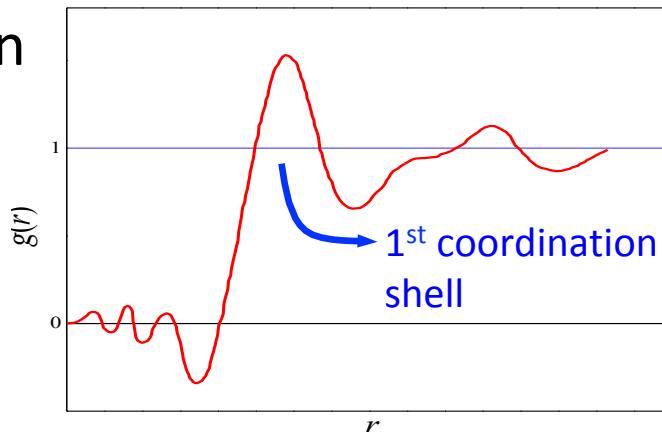
$S(\infty) = 1$

Fourier Transformation



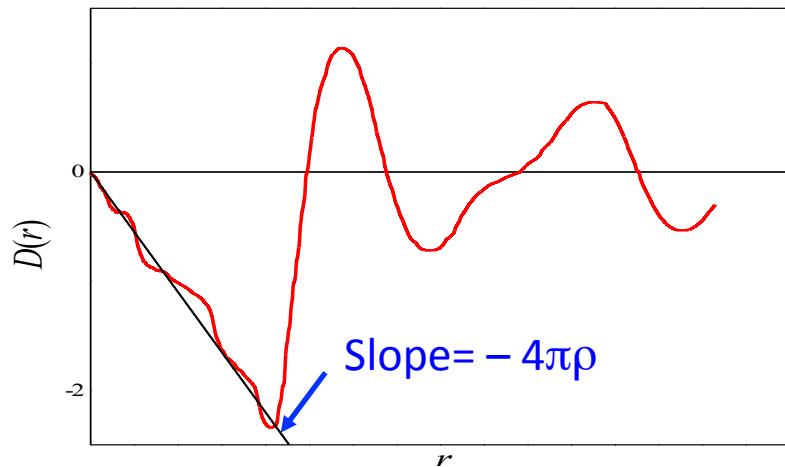
Related functions

Pair distribution function $g(r)$



Pair correlation function $G(r)$ or density function $D(r)$

$$G(r) = D(r) = 4\pi r \rho [g(r) - 1]$$



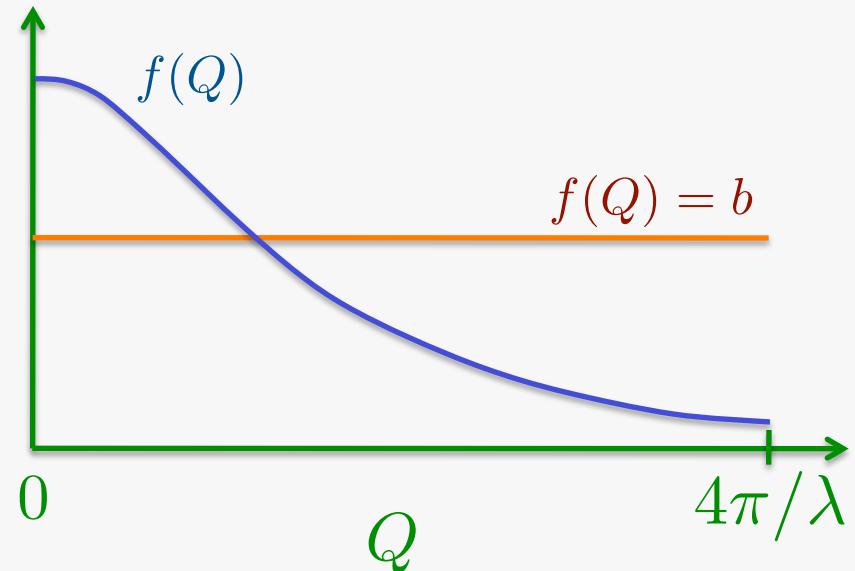
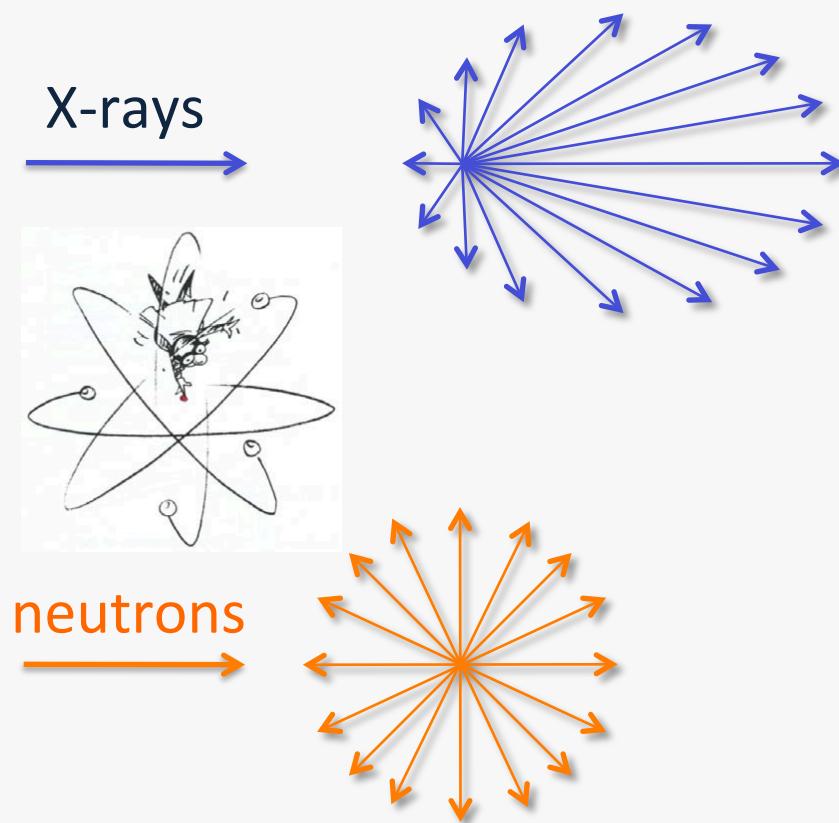
Remember!

$$g(r) - 1 \propto \text{FT} \{ S(Q) - 1 \} / \rho$$



X-rays vs neutrons

$$\left(\frac{d^2\sigma}{d\Omega dE'} \right) = \frac{k'}{k} \frac{1}{2\pi\hbar} \sum_{jj'} f_j(\vec{Q}) f_{j'}(\vec{Q}) \int_{-\infty}^{\infty} \langle e^{-i\vec{Q}\cdot\vec{r}_{j'}(0)} e^{i\vec{Q}\cdot\vec{r}_j(t)} \rangle e^{-i\omega t} dt$$



Bragg law $Q = \frac{4\pi}{\lambda} \sin\left(\frac{2\theta}{2}\right)$



Multiaatomic systems

System of n chemical species

$$\bar{b}^2 \underbrace{[S(Q) - 1]}_{F(Q)} = \sum_{\alpha=1}^n \sum_{\beta=1}^n c_{\alpha} c_{\beta} b_{\alpha} b_{\beta} [S_{\alpha\beta}(Q) - 1]$$

$n(n+1)/2$ independent partial $S_{\alpha\beta}(Q)$

Change b_{α} by

- Isotopic substitution
- X-ray experiments
- Anomalous diffraction

$$\bar{b}^2 = \sum_{\alpha=1}^n \sum_{\beta=1}^n c_{\alpha} c_{\beta} b_{\alpha} b_{\beta}$$

$$\mathbf{F}_{\text{exp}}(Q) = \mathbf{A} \mathbf{F}_p(Q)$$

NDIS: $|\mathbf{A}| < 0.1$

Binary system:

Two different species: x, y

Fixed composition: constant c_x, c_y

Isotopes with good contrast

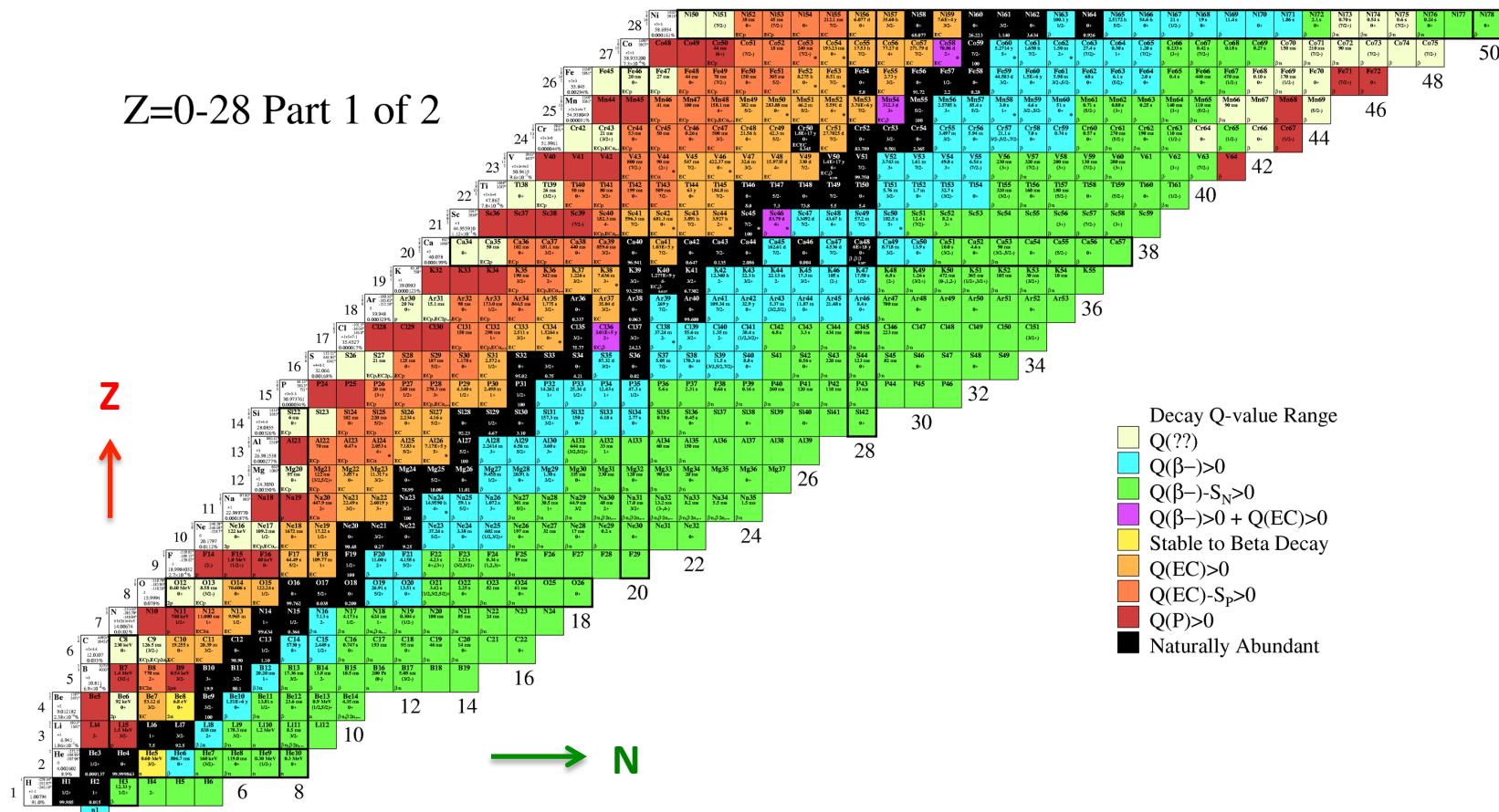
$b_{\alpha i}$: scattering length of isotope i of species α

$$\bar{b}^2 \begin{pmatrix} F_{S1}(Q) \\ F_{S2}(Q) \\ F_{S3}(Q) \end{pmatrix} = \begin{pmatrix} c_X^2 b_{X1}^2 & c_Y^2 b_{Y1}^2 & 2c_X c_Y b_{X1} b_{Y1} \\ c_X^2 b_{X2}^2 & c_Y^2 b_{Y2}^2 & 2c_X c_Y b_{X2} b_{Y2} \\ c_X^2 b_{X3}^2 & c_Y^2 b_{Y3}^2 & 2c_X c_Y b_{X3} b_{Y3} \end{pmatrix} \begin{pmatrix} F_{XX}(Q) \\ F_{YY}(Q) \\ F_{XY}(Q) \end{pmatrix}$$



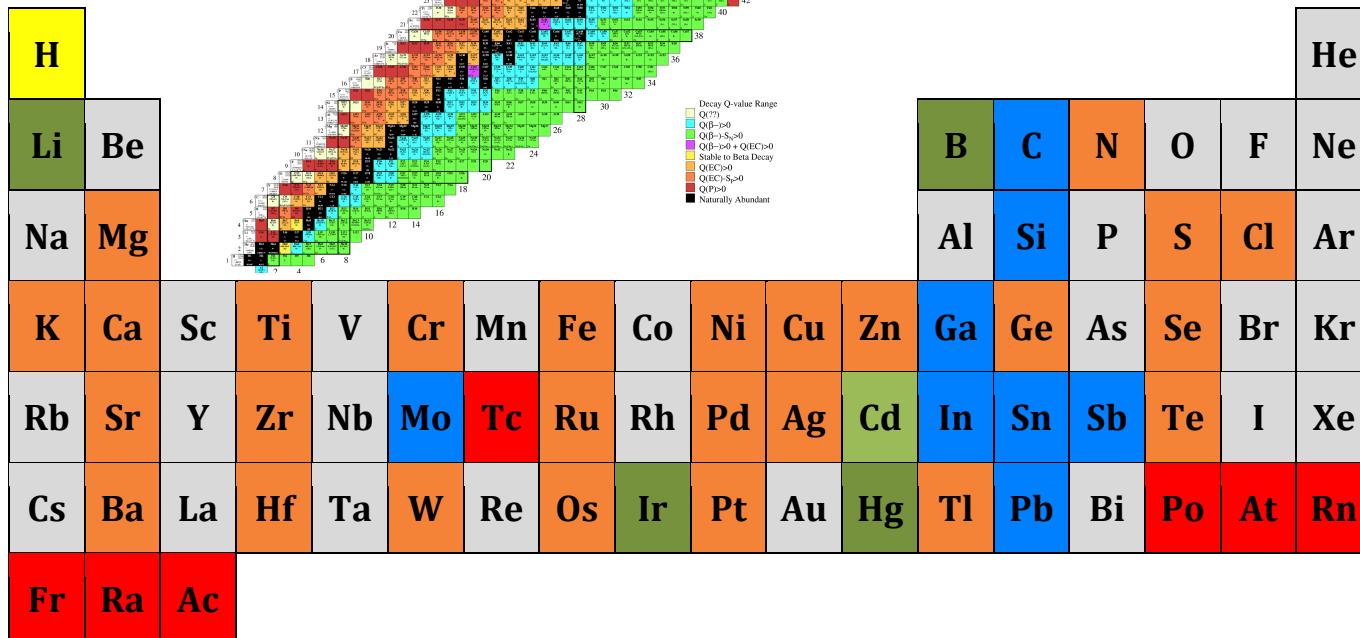
Isotopic substitution

Z=0-28 Part 1 of 2





Isotopic substitution



Appendix

Neutron Scattering Lengths and Cross Sections

Javier Dawidowski, José Rolando Granada, Javier Roberto Santisteban, Florencia Cantarigá and Luis Alberto Rodríguez Palomino
Comisión Nacional de Energía Atómica, Consejo Nacional de Investigaciones Científicas y Técnicas, Centro Atómico Bariloche and Instituto Balseiro, Bariloche, Río Negro, Argentina

Experimental Methods in the Physical Sciences, Vol. 44.
© 2013 Elsevier Inc. All rights reserved.

Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lw

Periodic table showing elements with isotopes with > 20 % scattering length contrast (orange), 5 - 20 % contrast (blue), mono-isotopic, lack of scattering length contrast or prohibitively expensive isotopes (grey), elements with high absorption coefficients where non-absorbing isotopes are available (green), elements with isotopes to overcome incoherent scattering effects (yellow) and radioactive elements (red).

20%

5%

1c\$

ABS

Abs

Inc

Rad

A binary system

Fast-ion conductor or semiconductor glasses

Silver chalcogenides



$\text{X} = \text{S, Te or Se}$

+

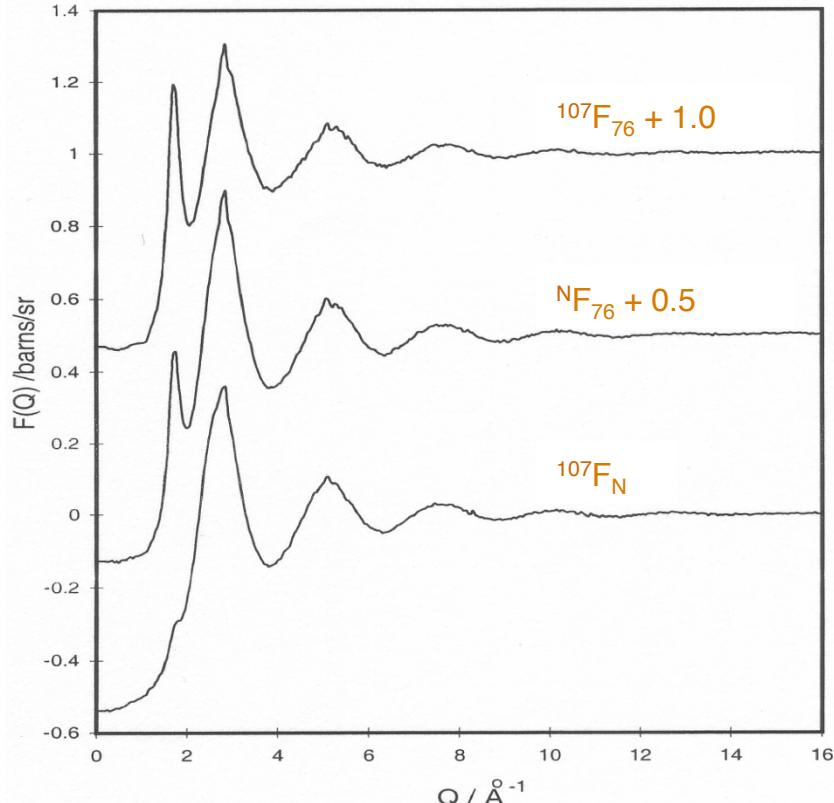
Network formers



Samples: $^{107}\text{Ag}_2^{\text{nat}}\text{Se}$, $^{109}\text{Ag}_2^{76}\text{Se}$, $^{\text{nat}}\text{Ag}_2^{76}\text{Se}$

Isotope	b (fm)	$\sigma_a(b)$	$\sigma_s(b)$
$^{\text{n}}\text{Ag}$	5.922	24.6	4.99
^{107}Ag	7.64	14.6	7.44
^{109}Ag	4.19	35.4	2.55
$^{\text{n}}\text{Se}$	7.97	4.55	8.31
^{76}Se	12.2	33.1	18.7

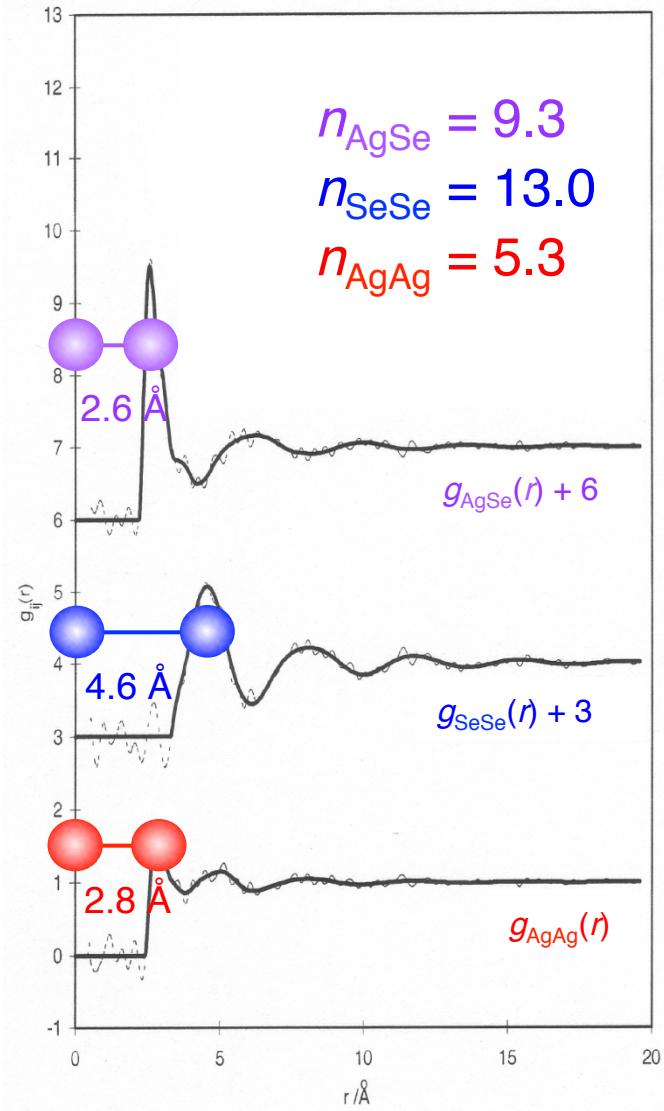
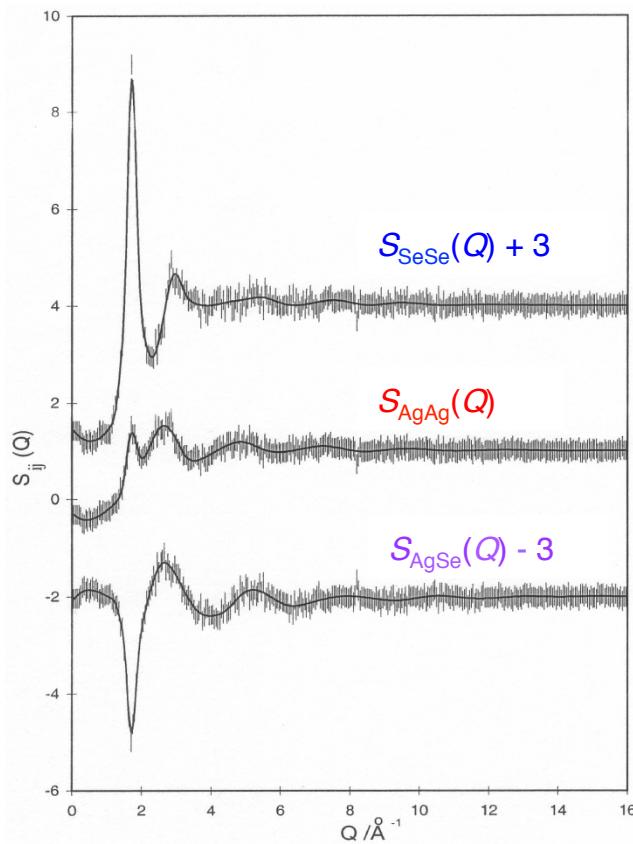
$$\begin{bmatrix} {}^{107}_{\text{nat}} F_{S1}(Q) \\ {}^{109}_{76} F_{S2}(Q) \\ {}^{\text{nat}}_{76} F_{S3}(Q) \end{bmatrix} = \begin{bmatrix} 0.2594 & 0.0706 & 0.2706 \\ 0.0780 & 0.1654 & 0.2272 \\ 0.1559 & 0.1654 & 0.3211 \end{bmatrix} \begin{bmatrix} F_{\text{AgAg}}(Q) \\ F_{\text{SeSe}}(Q) \\ F_{\text{AgSe}}(Q) \end{bmatrix}$$





Partial structure factors

$$\begin{bmatrix} F_{\text{AgAg}}(Q) \\ F_{\text{SeSe}}(Q) \\ F_{\text{AgSe}}(Q) \end{bmatrix} = \begin{bmatrix} 12.17 & 17.31 & -22.50 \\ 8.11 & 32.22 & -29.63 \\ -10.09 & -25.00 & 29.30 \end{bmatrix} \begin{bmatrix} {}^{107}_{\text{nat}} F_{S1}(Q) \\ {}^{109}_{\text{nat}} F_{S2}(Q) \\ {}^{107}_{\text{nat}} F_{S3}(Q) \end{bmatrix}$$



Ionic behaviour





First difference method

$$\bar{b}^2 F(Q) = \sum_{\alpha=1}^n \sum_{\beta=1}^n c_\alpha c_\beta b_\alpha b_\beta F_{\alpha\beta}(Q)$$

Substitution $\longrightarrow \gamma: \gamma_1, \gamma_2$

Important! We change scattering lengths but not composition

$$\bar{b}^2 F_{\gamma 1}(Q) = c_\gamma^2 b_{\gamma 1}^2 F_{\gamma\gamma}(Q) + c_\gamma b_{\gamma 1} \sum_{\alpha \neq \gamma} c_\alpha b_\alpha F_{\alpha\gamma}(Q) + \sum_{\alpha, \beta \neq \gamma} c_\alpha c_\beta b_\alpha b_\beta F_{\alpha\beta}(Q)$$

$$\bar{b}^2 F_{\gamma 2}(Q) = c_\gamma^2 b_{\gamma 2}^2 F_{\gamma\gamma}(Q) + c_\gamma b_{\gamma 2} \sum_{\alpha \neq \gamma} c_\alpha b_\alpha F_{\alpha\gamma}(Q) + \sum_{\alpha, \beta \neq \gamma} c_\alpha c_\beta b_\alpha b_\beta F_{\alpha\beta}(Q)$$

Correlation
function of
atom γ with
all other
components

$$\bar{b}^2 \Delta F_\gamma(Q) = c_\gamma^2 (b_{\gamma 1}^2 - b_{\gamma 2}^2) \boxed{F_{\gamma\gamma}(Q)} + c_\gamma (b_{\gamma 1} - b_{\gamma 2}) \sum_{\alpha \neq \gamma} c_\alpha b_\alpha \boxed{F_{\alpha\gamma}(Q)}$$

$$\frac{\bar{b}^2 \Delta F_\gamma(Q)}{c_\gamma^2 (b_{\gamma 1}^2 - b_{\gamma 2}^2)} = F_{\gamma\gamma}(Q) + \frac{\sum_{\alpha \neq \gamma} c_\alpha b_\alpha F_{\alpha\gamma}(Q)}{c_\gamma (b_{\gamma 1} + b_{\gamma 2})} \xrightarrow{\text{small}}$$



A ternary system

Li in ND₃

Metal-nonmetal transition at 7 MPM

Class A metals

Conductivity $15000 \Omega^{-1} \text{ cm}^{-1} \text{ mol}^{-1}$

3 species \Rightarrow 6 different experiments!

First difference method

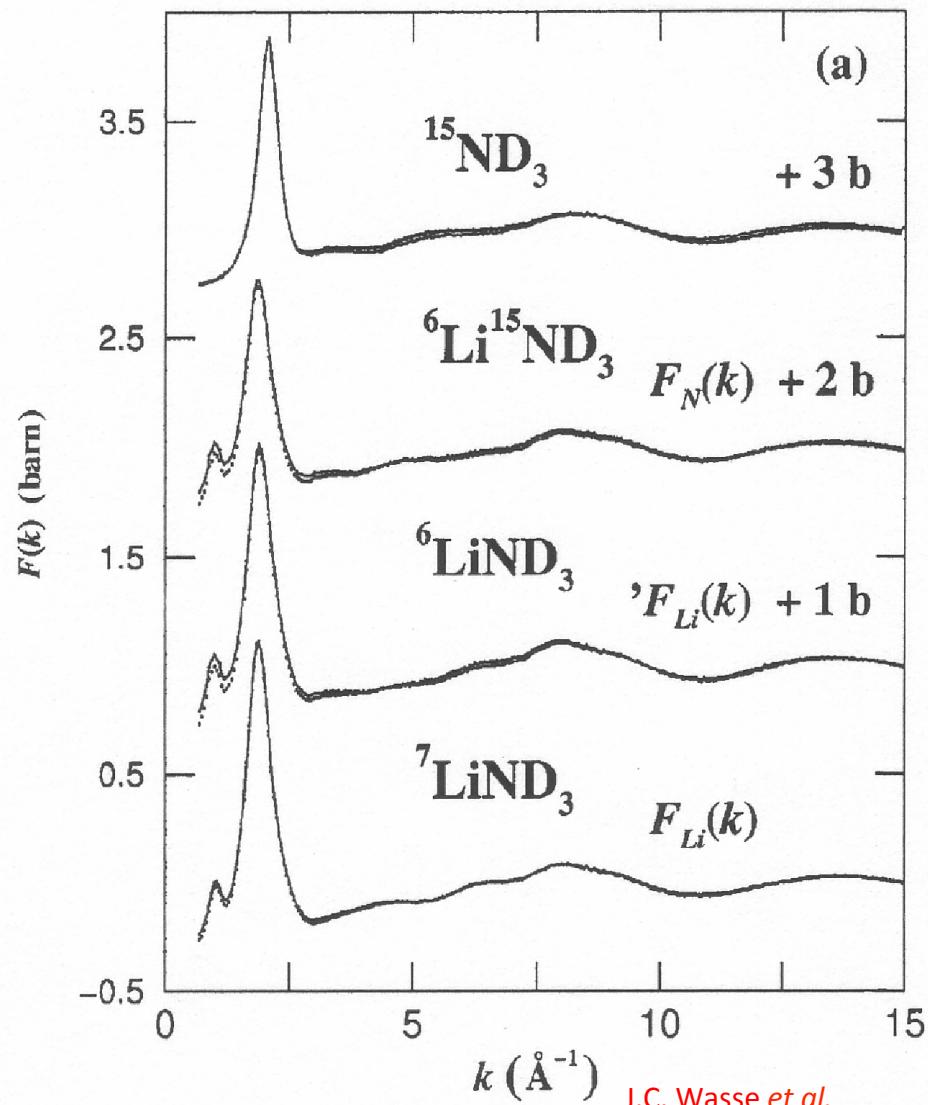
Samples:

^6Li in $^{nat}\text{ND}_3$

^7Li in $^{nat}\text{ND}_3$

^6Li in $^{15}\text{ND}_3$

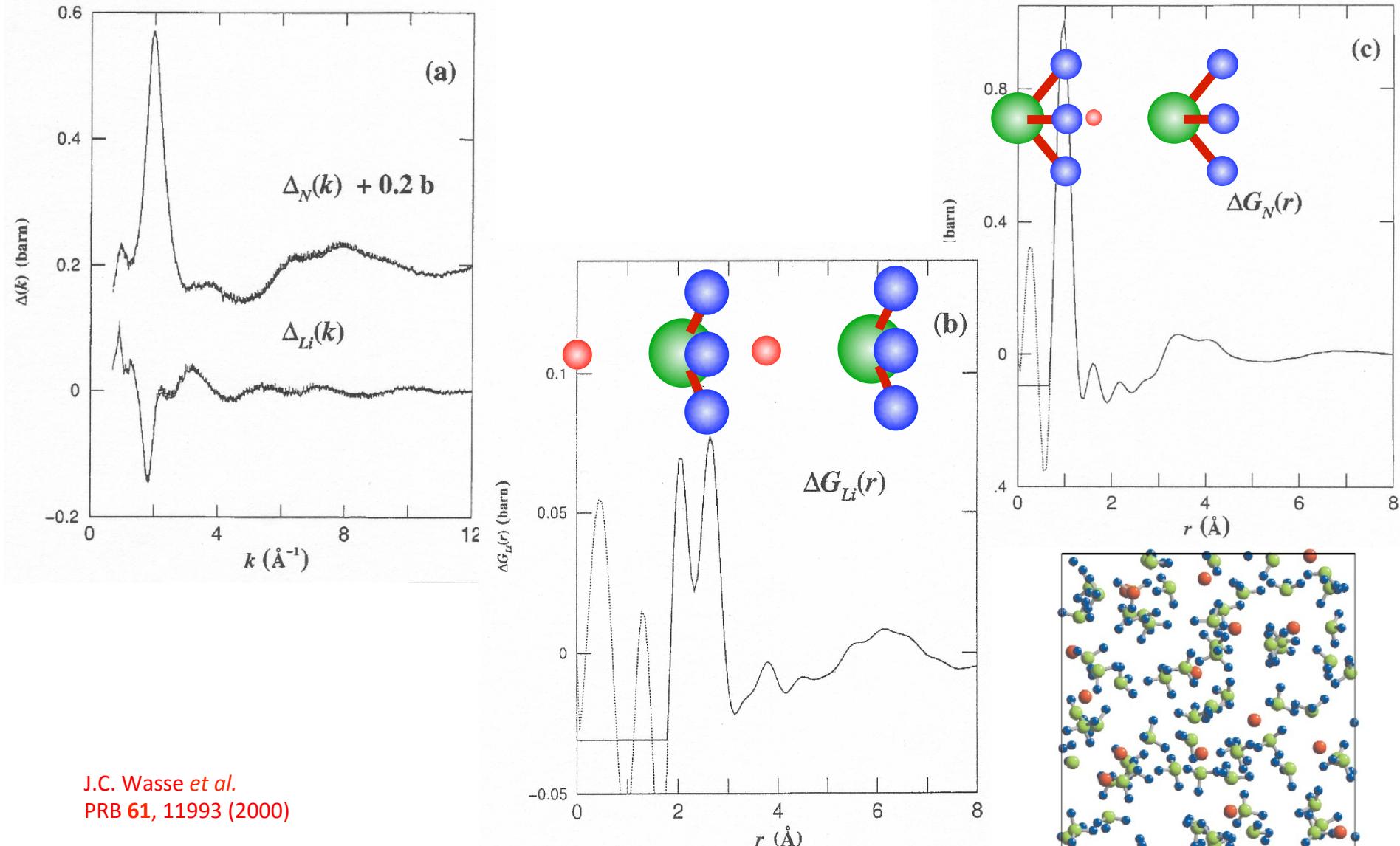
$^{15}\text{ND}_3$



J.C. Wasse *et al.*
PRB **61**, 11993 (2000)



Quasi-partial





Second difference method

New substitution $\delta: \delta_1, \delta_2$

$$\bar{b}^2 \Delta F_{\gamma\delta 1}(Q) = c_\gamma^2(b_{\gamma 1}^2 - b_{\gamma 2}^2) F_{\gamma\gamma}(Q) + c_\gamma c_\delta(b_{\gamma 1} - b_{\gamma 2}) b_{\delta 1} F_{\gamma\delta}(Q) + c_\gamma(b_{\gamma 1} - b_{\gamma 2}) \sum_{\alpha \neq \gamma, \delta}^n c_\alpha b_\alpha F_{\alpha\gamma}(Q)$$

$$\bar{b}^2 \Delta F_{\gamma\delta 2}(Q) = c_\gamma^2(b_{\gamma 1}^2 - b_{\gamma 2}^2) F_{\gamma\gamma}(Q) + c_\gamma c_\delta(b_{\gamma 1} - b_{\gamma 2}) b_{\delta 2} F_{\gamma\delta}(Q) + c_\gamma(b_{\gamma 1} - b_{\gamma 2}) \sum_{\alpha \neq \gamma, \delta}^n c_\alpha b_\alpha F_{\alpha\gamma}(Q)$$

$$\bar{b}^2 \Delta^2 F_{\gamma\delta}(Q) = c_\gamma c_\delta(b_{\gamma 1} - b_{\gamma 2})(b_{\delta 1} - b_{\delta 2}) F_{\gamma\delta}(Q)$$

$$F_{\gamma\delta}(Q) = \frac{\bar{b}^2 \Delta^2 F_{\gamma\delta}(Q)}{c_\gamma c_\delta(b_{\gamma 1} - b_{\gamma 2})(b_{\delta 1} - b_{\delta 2})}$$

Partial structure factor
for pairs γ and δ

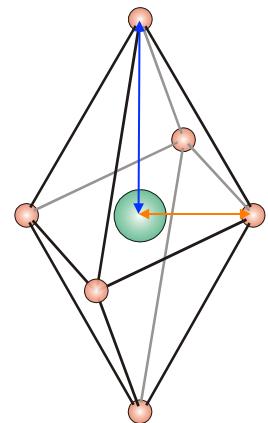


Cu(II) aqua ion

A. Pasquarello *et al.*
Science 291, 856 (2001)

Model

Octahedral complex $[\text{Cu}(\text{H}_2\text{O})]^{2+}$
Sixfold coordination



X-ray diffraction
EXAFS
XANES
NDIS

A priori assumptions about structure

Overlap axial Cu-O and Cu-H

Second difference method

$$\Delta F_H = c_{\text{Cu}}^2 (b_{65}^{-2} - b_{63}^{-2}) F_{\text{CuCu}} + 2 c_{\text{Cu}} (b_{65} - b_{63}) \times (c_{\text{Cl}} b_{\text{Cl}} F_{\text{CuCl}} + c_{\text{O}} b_{\text{O}} F_{\text{CuO}} + c_{\text{H}} b_{\text{H}} F_{\text{CuH}})$$

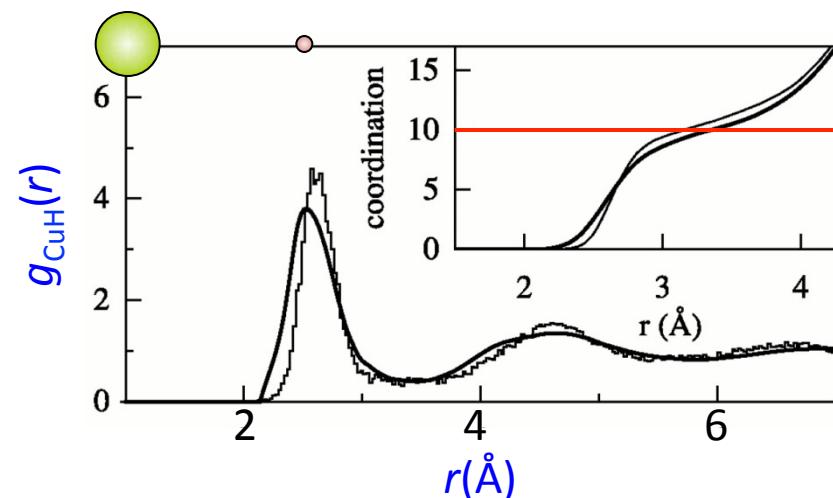
$$\Delta^2 F = 2 c_{\text{Cu}} c_{\text{H}} (b_{65} - b_{63}) (b_{\text{D}} - b_{\text{H}}) F_{\text{CuH}}$$

System:

$\text{Cu}(\text{ClO}_4)_2 + \text{HClO}_4$ in H_2O
→ 10 expts!

Samples:

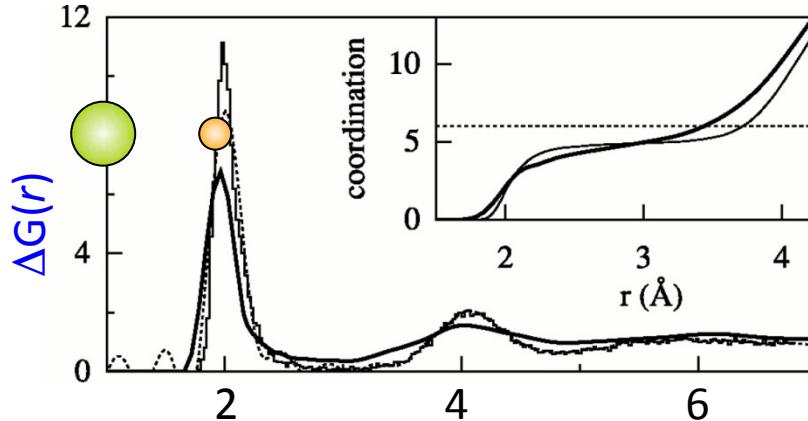
$^{65}\text{Cu}(\text{ClO}_4)_2 + \text{HClO}_4$ in H_2O
 $^{63}\text{Cu}(\text{ClO}_4)_2 + \text{HClO}_4$ in H_2O
 $^{65}\text{Cu}(\text{ClO}_4)_2 + \text{DClO}_4$ in D_2O
 $^{63}\text{Cu}(\text{ClO}_4)_2 + \text{DClO}_4$ in D_2O





Five-fold coordinated ion

A. Pasquarello *et al.*
Science 291, 856 (2001)

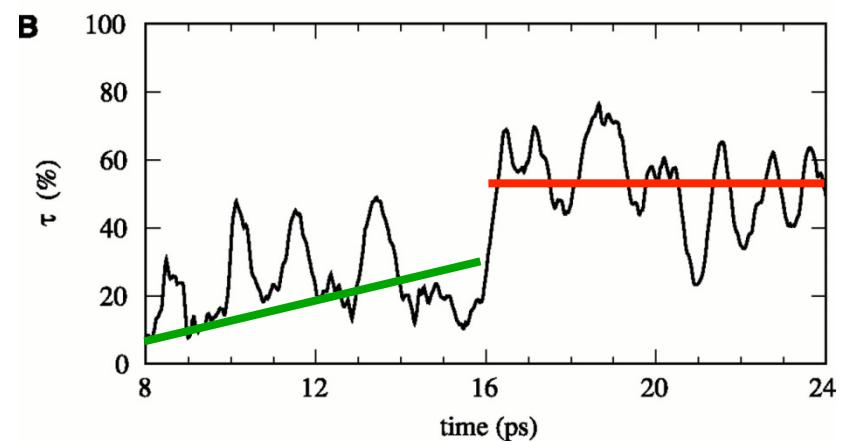
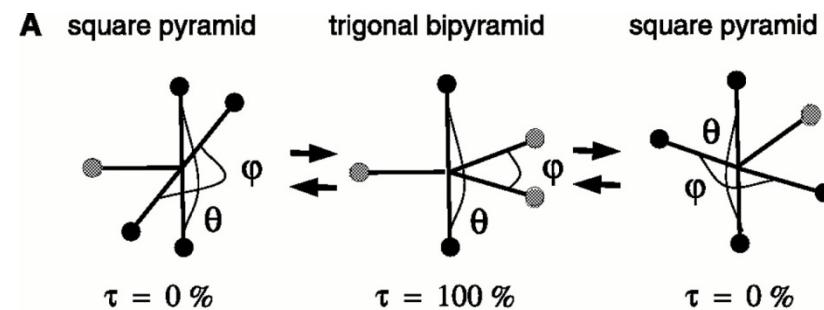


First-principles Molecular
Dynamics Simulation

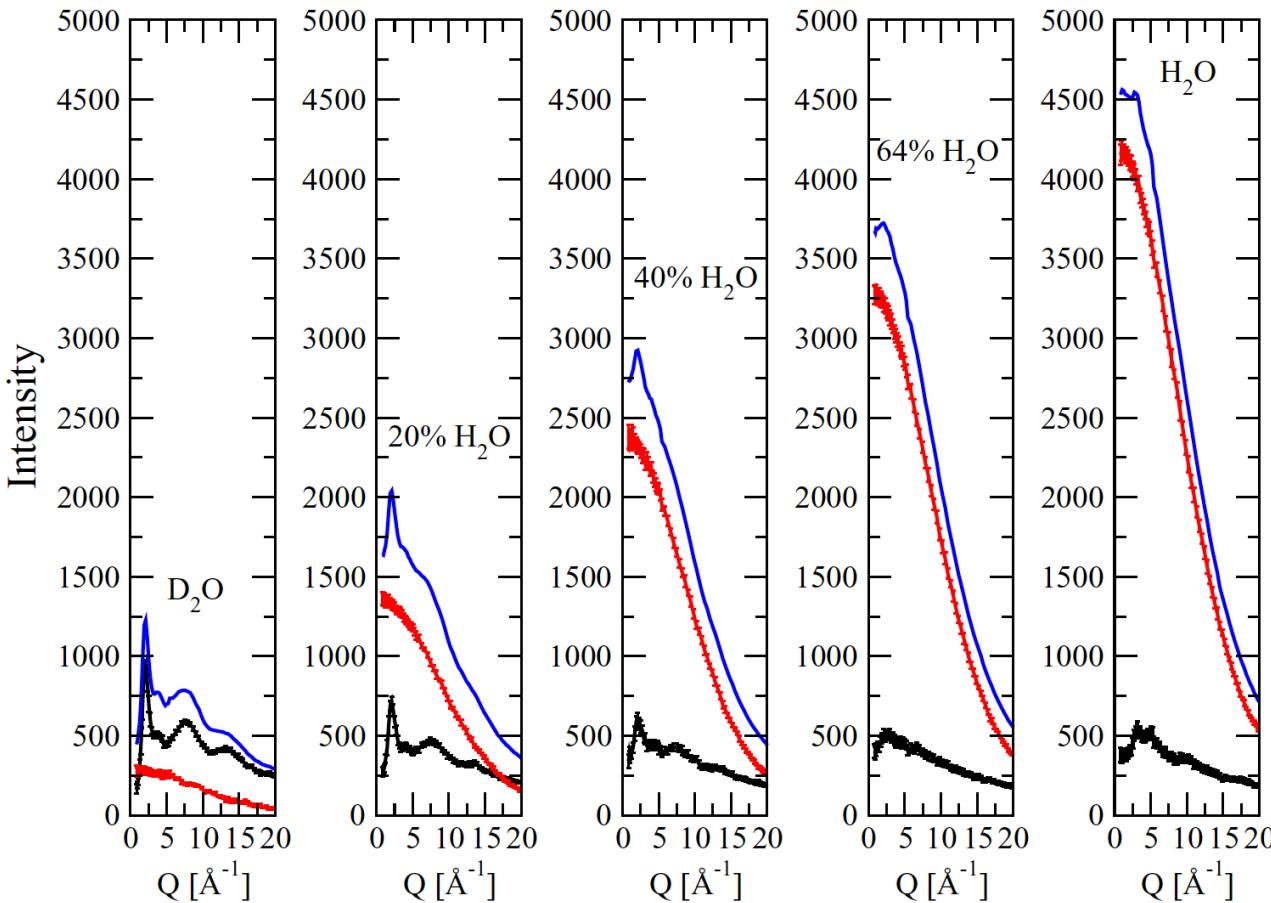
$$\tau = (\theta - \varphi)/60 \times 100\%$$

Cu(II) aqua ion is
five-fold coordinated

$$\Delta F = F_{\text{CuO}} + 0.044 F_{\text{CuCu}} + 0.102 F_{\text{CuCl}}$$

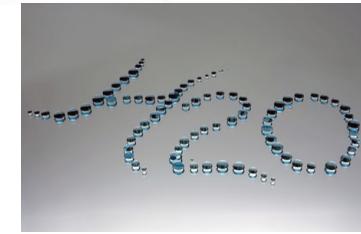


H incoherence problem



Neutron diffraction of hydrogenous materials: Measuring incoherent and coherent intensities separately

László Temleitner, Anne Stunault, Gabriel J. Cuello, and László Pusztai
Phys. Rev. B **92**, 014201 – Published 1 July 2015



Isotopes

σ_c	σ_i	σ_s	σ_a
44.89 (4)	0	44.89 (4)	0
1.7568 (10)	80.26(6)	82.02 (6)	0.3326 (7)
1.7589 (11)	79.91(4)	81.67 (4)	0.3326 (7)
5.597 (10)	2.04(3)	7.64(3)	0.000519 (7)
2.89(3)	0.14(4)	3.03(5)	<6E-06

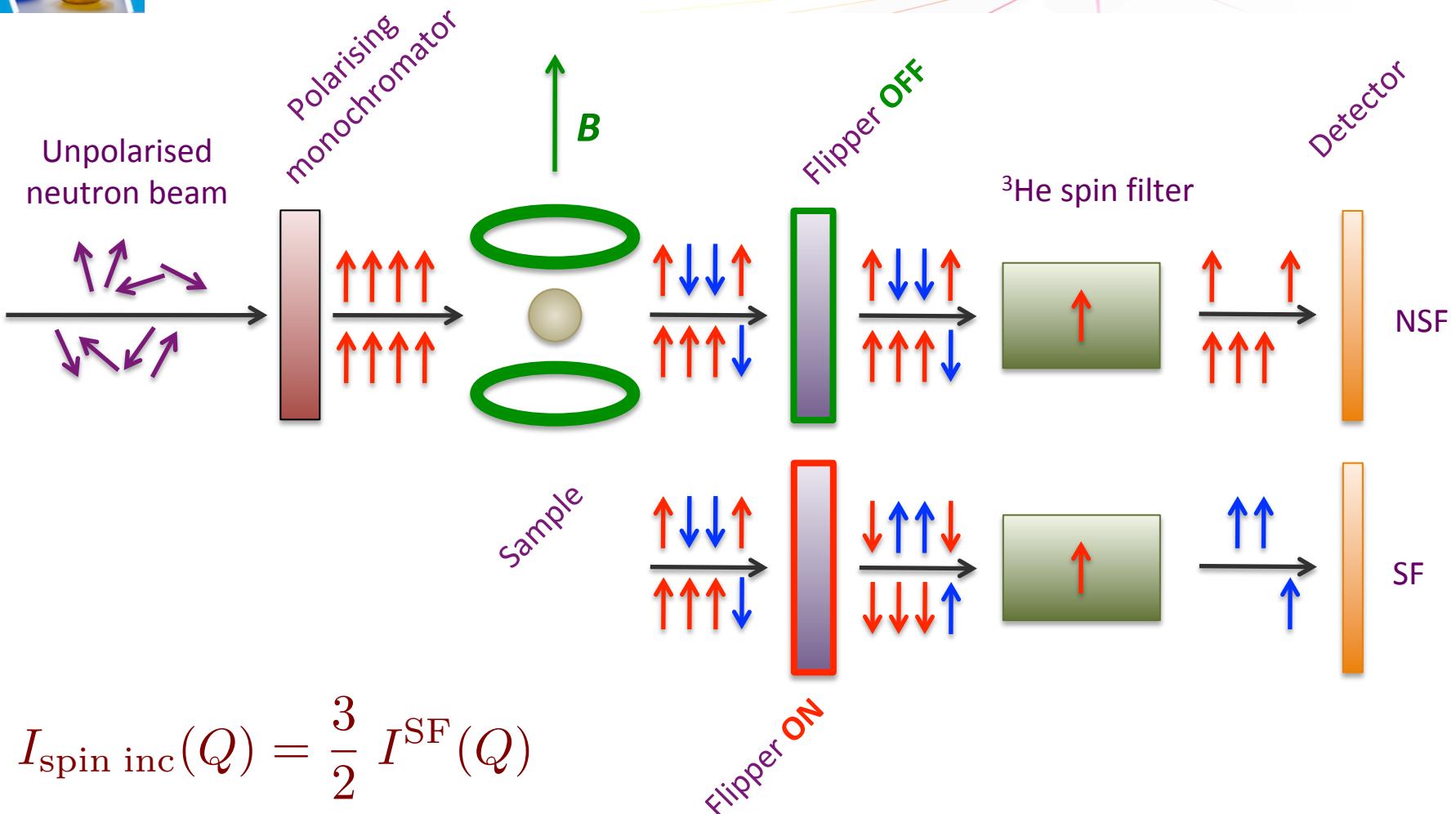
Appendix

Neutron Scattering Lengths and Cross Sections

Javier Dawidowski, José Rolando Granada, Javier Roberto Santisteban, Florencia Cantargi and Luis Alberto Rodríguez Palomino
Comisión Nacional de Energía Atómica, Consejo Nacional de Investigaciones Científicas y Técnicas, Centro Atómico Bariloche and Instituto Balseiro, Bariloche, Río Negro, Argentina

Experimental Methods in the Physical Sciences, Vol. 44.
© 2013 Elsevier Inc. All rights reserved.

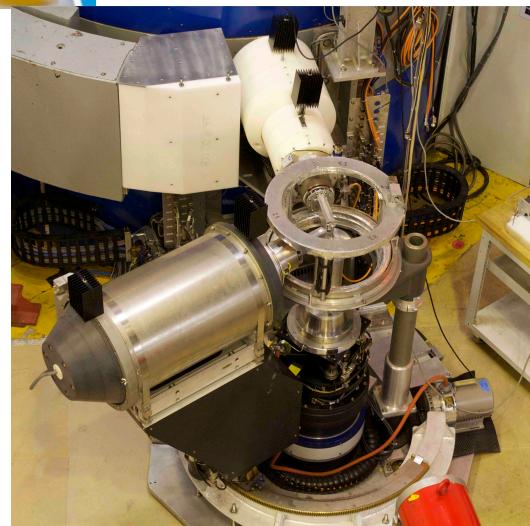
Polarised neutrons



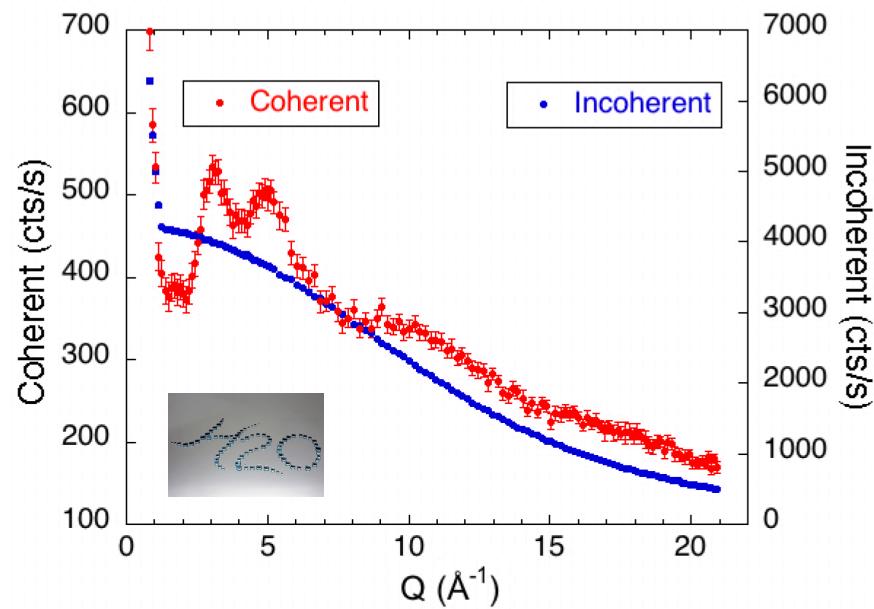
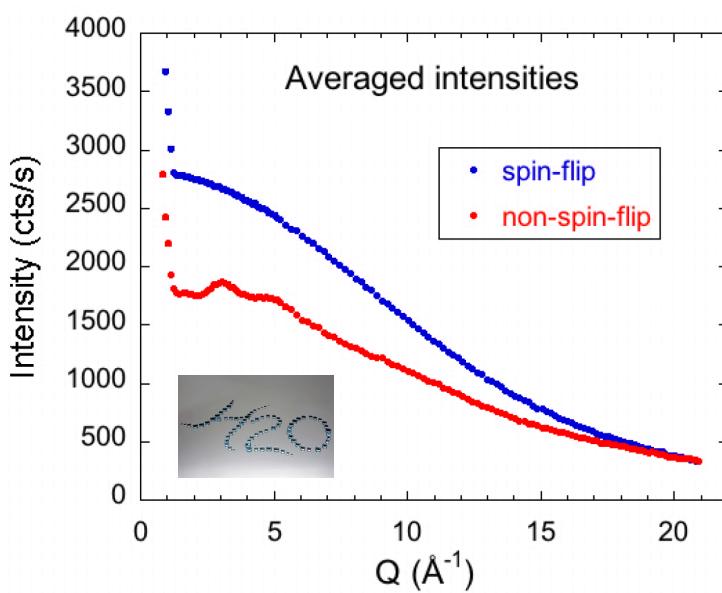
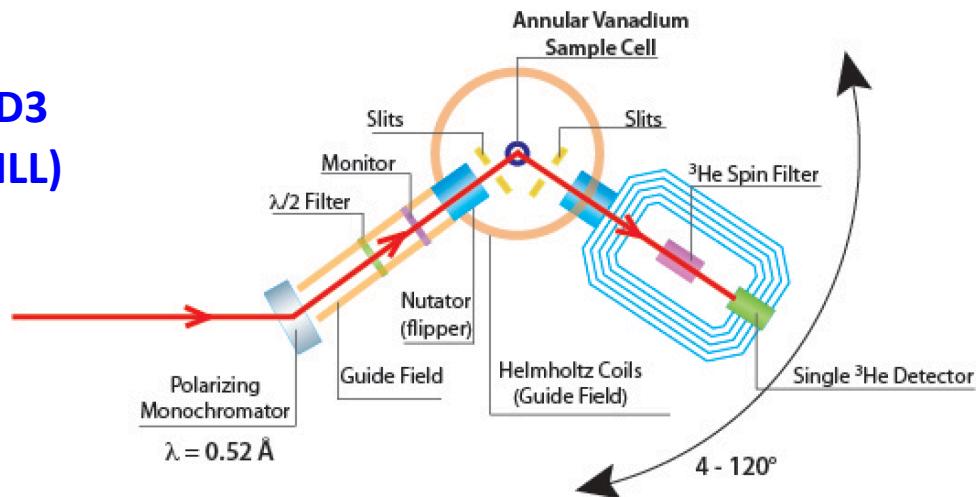
$$I_{\text{spin inc}}(Q) = \frac{3}{2} I^{\text{SF}}(Q)$$

$$I_{\text{coh}}(Q) + I_{\text{isotope inc}}(Q) = I^{\text{NSF}}(Q) - \frac{1}{2} I^{\text{SF}}(Q)$$

Example: water



D3
(ILL)





How to assess the structure of glasses ?

CNRS thematic school about glass structure



Tutorial: Neutron total scattering

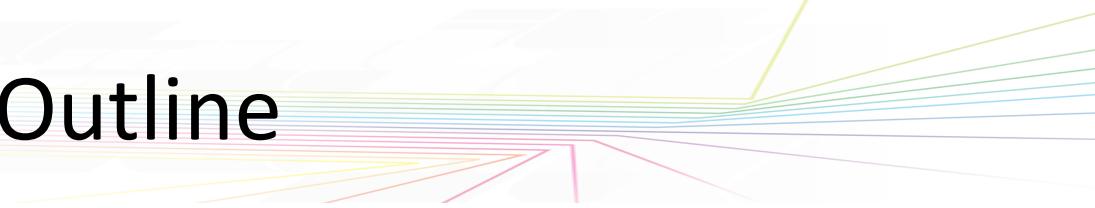
Gabriel Cuello
Institut Laue Langevin
Grenoble, France

cuello@ill.eu

Characterization of glass structure
18 - 22 November 2019
EPN Campus – Grenoble, France



Outline



- Instruments
- Raw data
- Background subtraction
- Multiple scattering
- Inelasticity effects
- Normalisation to absolute scale
- Fourier transformation

Instruments

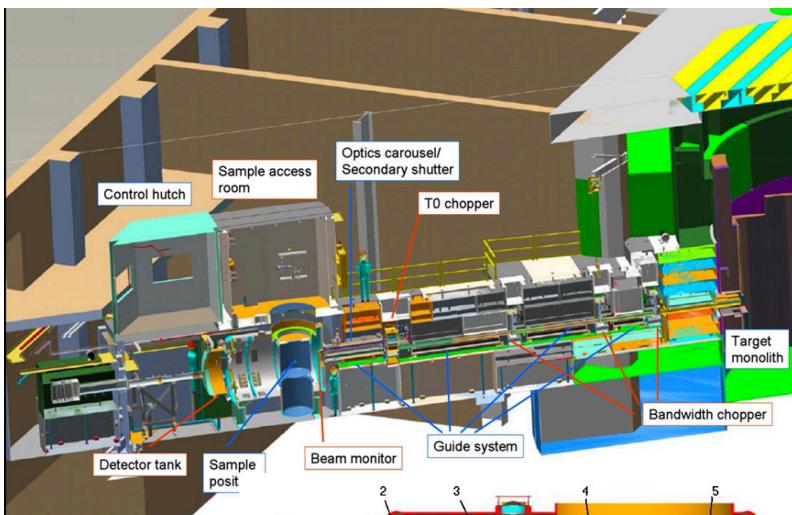
Reactor → 2-axis

→ Scattering angle

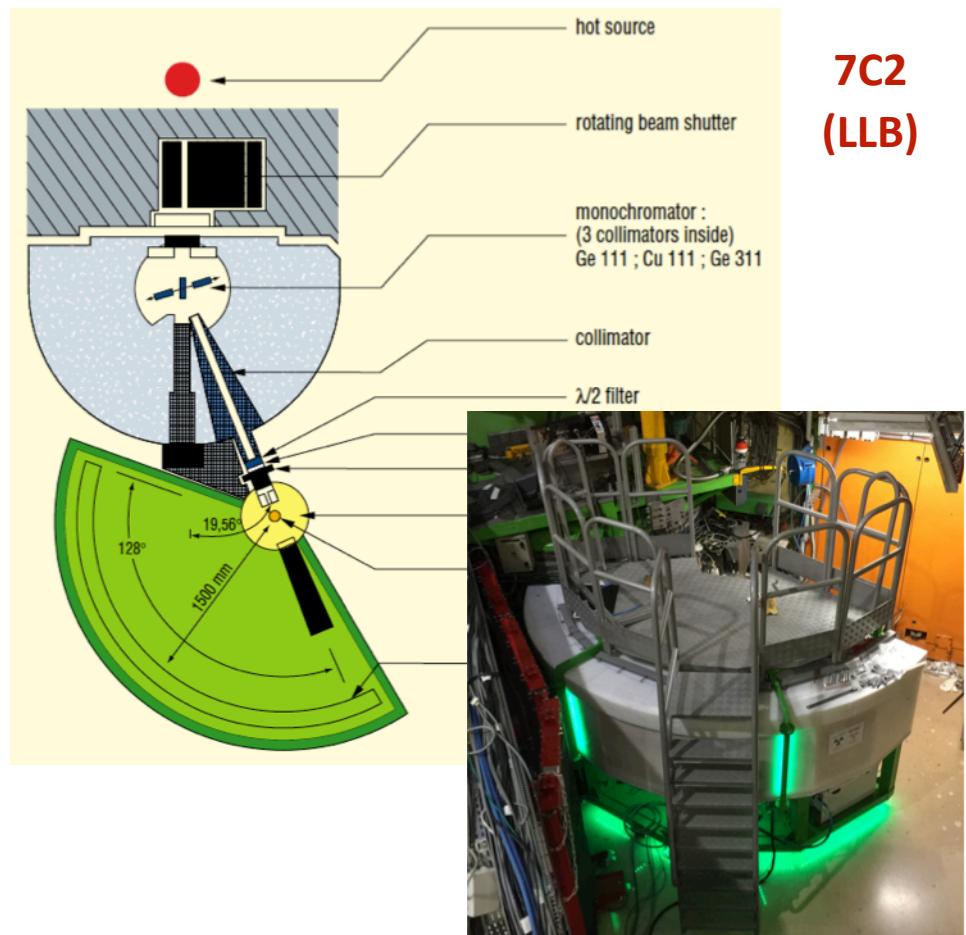
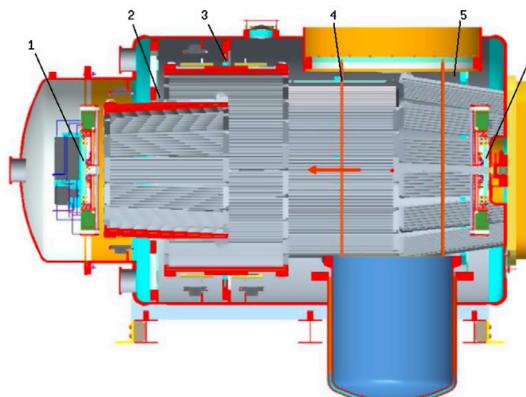
Accelerator → TOF

→ Time-of-flight

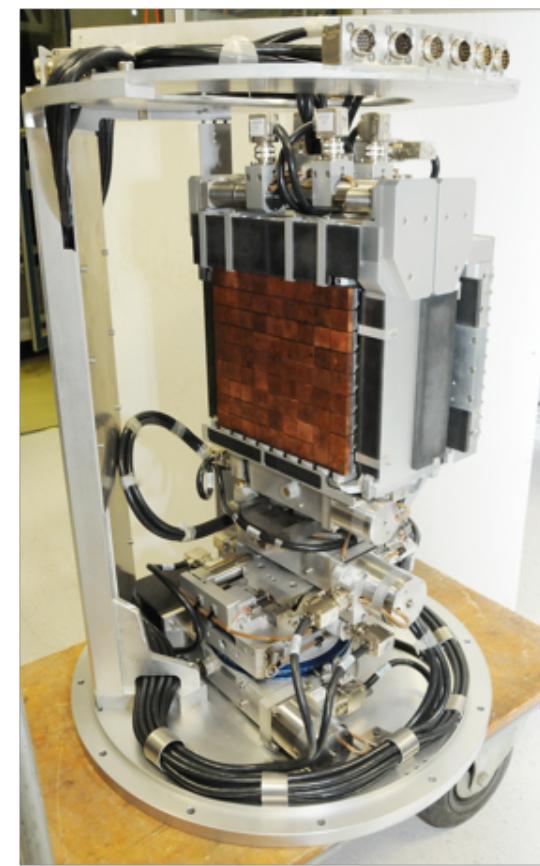
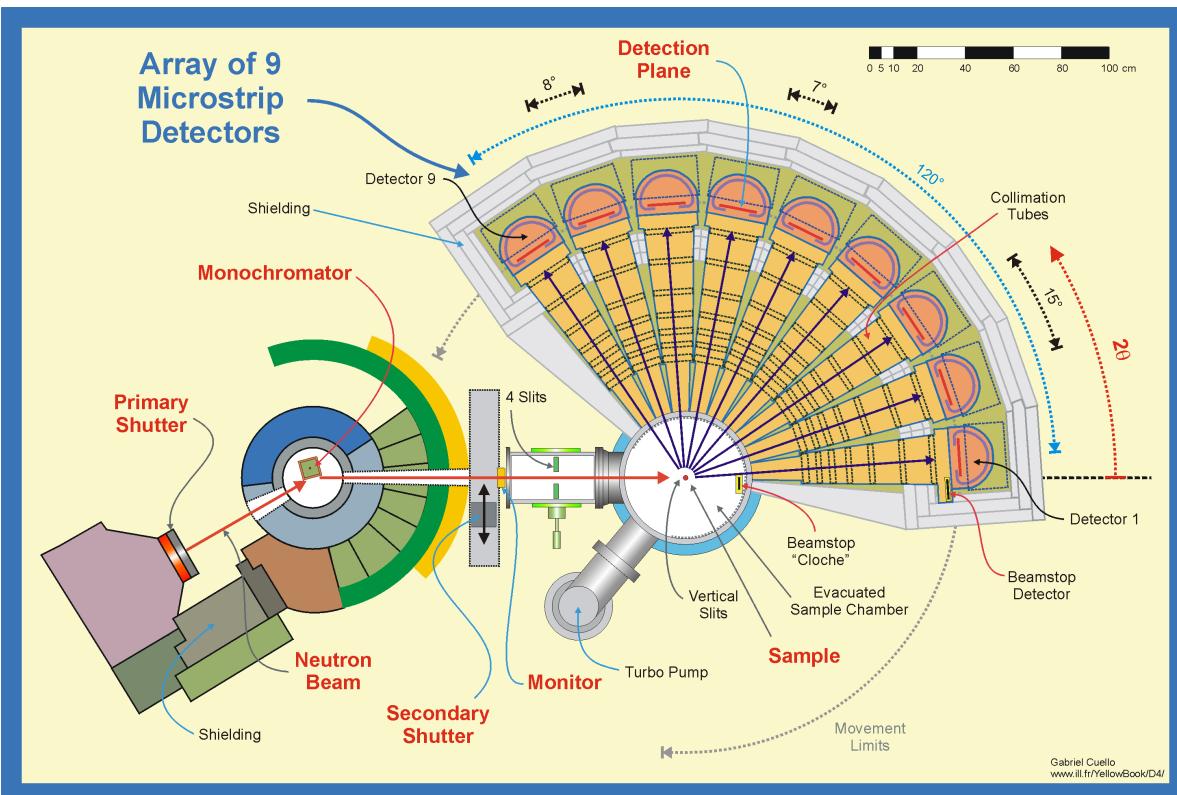
Elastic scattering → Q



**NOMAD
(ORNL)**



D4C @ ILL

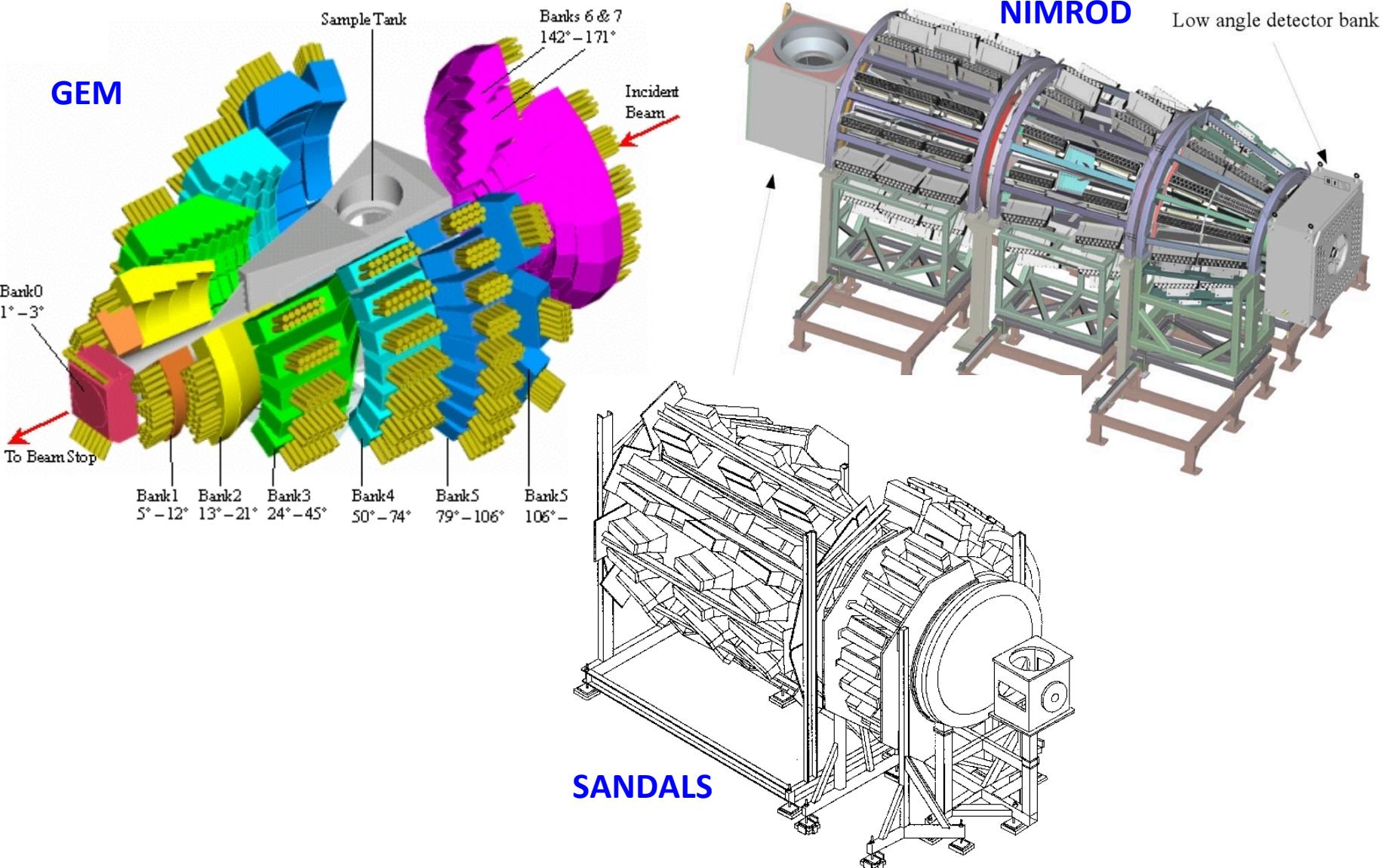


Face	d-spacing (Å)	λ (Å)	Flux ($10^7 \text{ n cm}^{-2} \text{ s}^{-1}$)	Filter
Si111	1.807	0.7	5.0	Ir
Cu220	1.278	0.5	4.5	Rh
Cu331	0.829	0.35	0.3	Non

$$Q = \frac{4\pi}{\lambda} \sin\left(\frac{2\theta}{2}\right)$$



Instruments @ ISIS

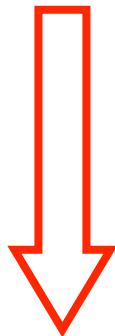




Data reduction

$$I(2\theta, \omega) = C \Phi_0 N \frac{k'}{k} \frac{\sigma}{4\pi} S(\vec{Q}, \omega) \epsilon(k')$$

$$I(2\theta) = C \Phi_0 N \frac{\sigma}{4\pi} \int_{-\infty}^{E_{\max}} d\omega \frac{k'}{k} S(\vec{Q}, \omega) \epsilon(k')$$



Formal aspects

- Elastic scattering (diffraction)
 - Stationary beam
 - Constant efficiency detector
 - One interaction processes (single scattering)
- Bragg's law (for Q)
Integration limits ($\pm\infty$)

$$I(2\theta) = C \Phi_0 N / 4\pi (\sigma_{\text{coh}} S(Q) + \sigma_{\text{inc}}) \epsilon(k)$$

Practical aspects

- Monochromatic beam
- No background
- No attenuation
- Single scattering



- No beam
- No container
- No sample
- No environment
- No detector



No problem!

Final analysis

$I_{\text{exp}}(2\theta)$

Corrections

$S(Q)$

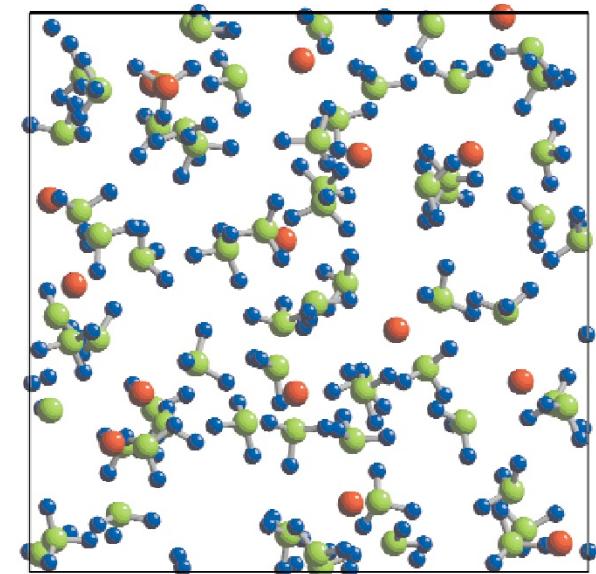
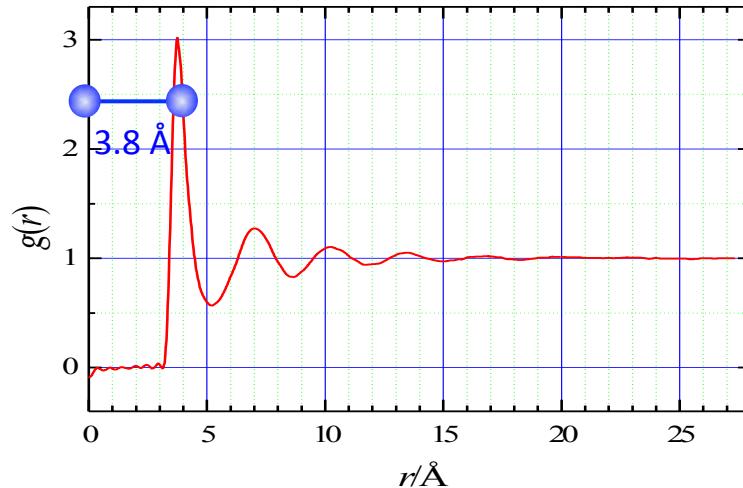
FT

$g(r)$

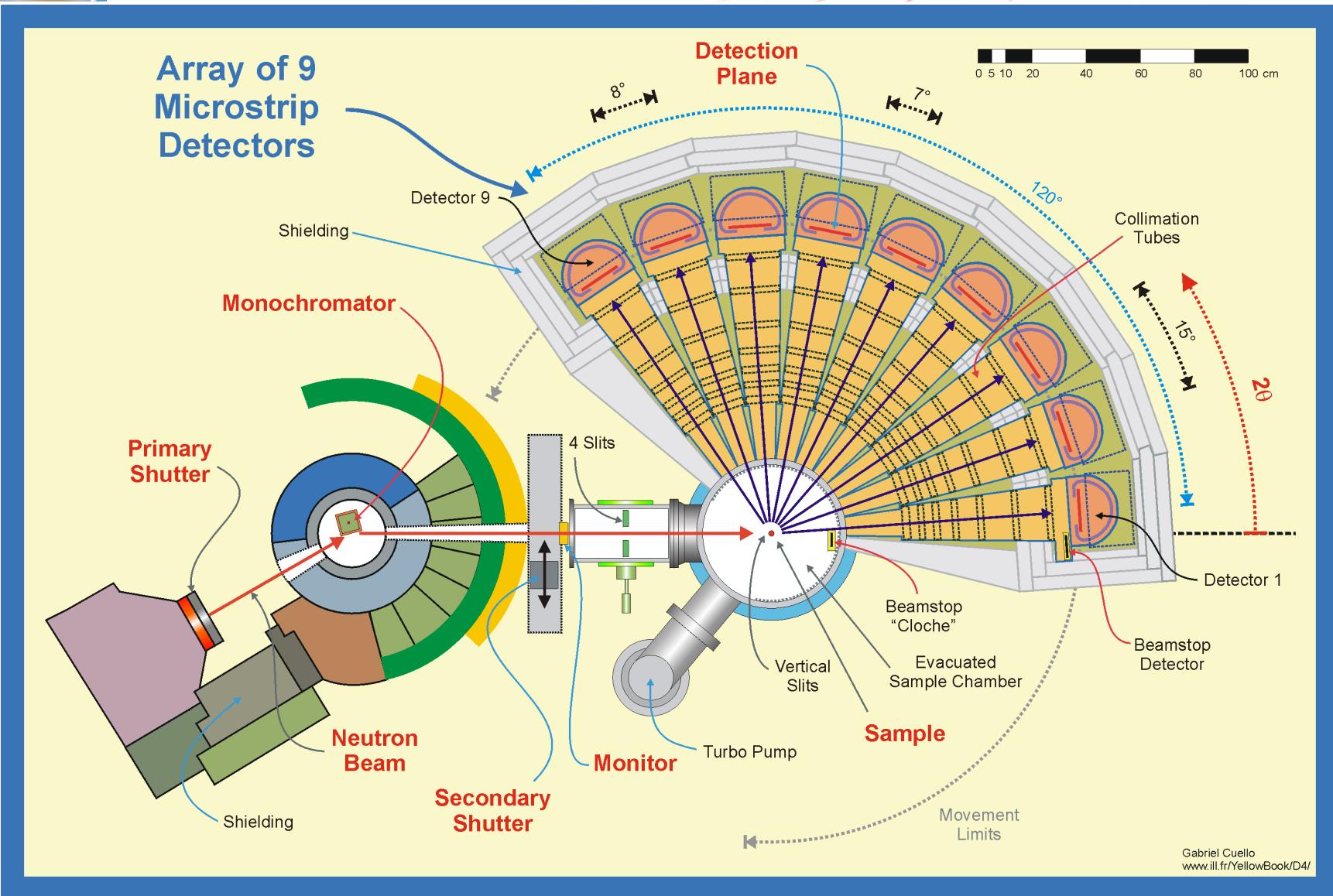
Truncation effects
Window functions

Interpretation

{ Model
Simulation, e.g. RMC, EPSR, MD, etc.



Instrument (D4@ILL)





Heavy atoms

SeTe alloys: different compositions and temperatures

We have measured 3 samples with compositions $\text{Se}_x\text{Te}_{1-x}$ with $x = 0.15, 0.20 and 0.25 at different temperatures:$

$\text{Se}_{15}\text{Te}_{85}$: $T = 425$ C, 485 C and 550 C.

$\text{Se}_{20}\text{Te}_{80}$: $T = 380$ C, 400 C, 450 C, 550 C and 650 C.

$\text{Se}_{25}\text{Te}_{75}$: $T = 400$ C, 520 C and 650 C.

Quartz sample containers, at $T = 350$ C, 450 C, 550 C and 650 C.

We assume we have two diffractograms D_1 and D_2 measured at the temperatures T_1 and T_2 , respectively, with $T_2 > T > T_1$.

The interpolated diffractogram is then

$$D = f_1 D_1 + f_2 D_2$$

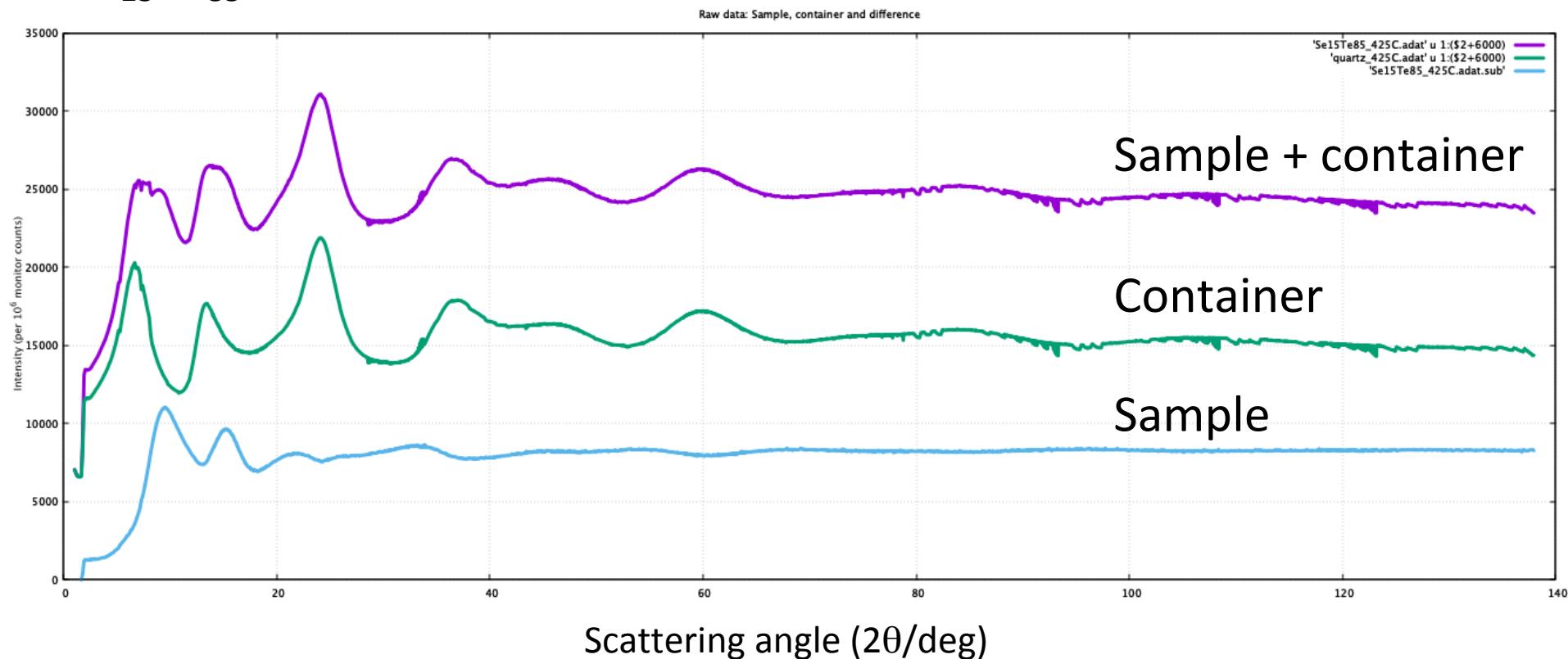
where

$$f_1 = (T_2 - T)/(T_2 - T_1) \quad f_2 = (T - T_1)/(T_2 - T_1).$$



Raw data

$\text{Se}_{15}\text{Te}_{85}$ @ 425 C

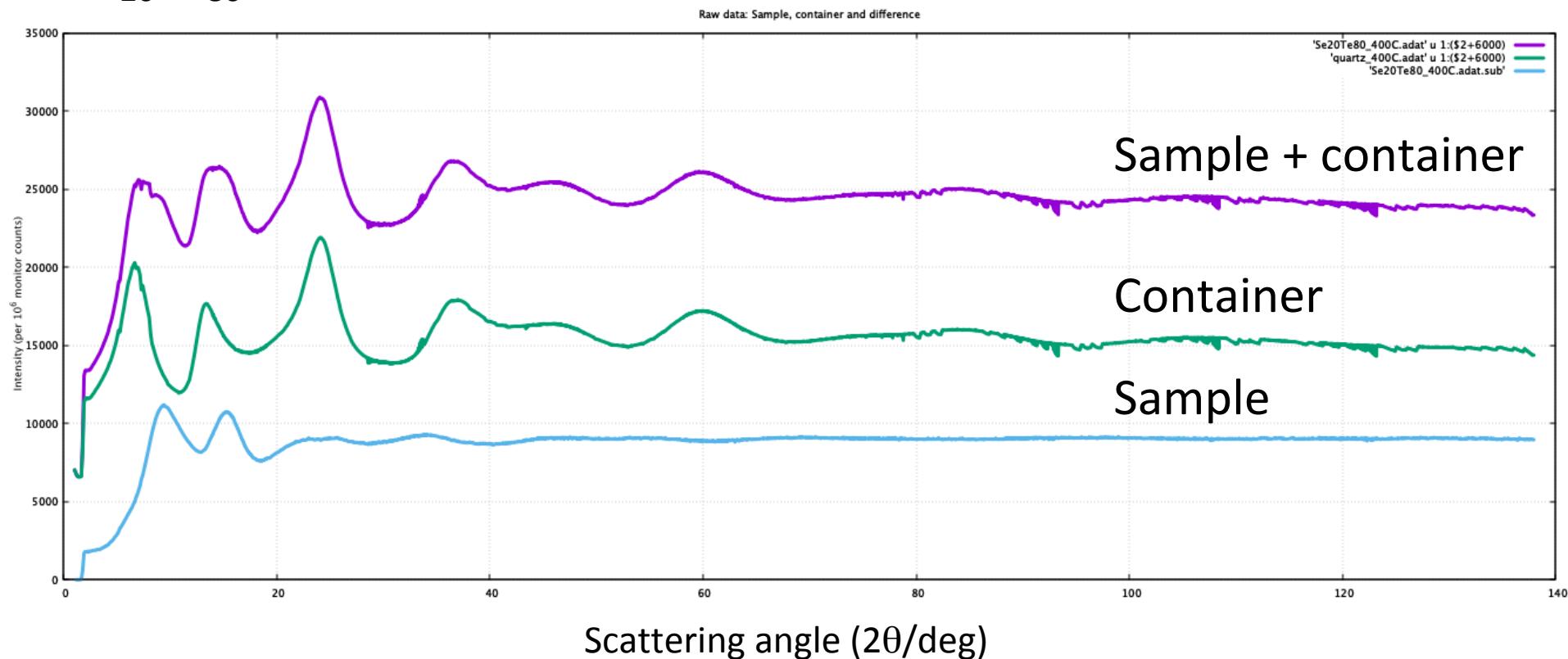


$$\text{Sample} = (\text{Sample+container}) - \text{Container}$$



Raw data

$\text{Se}_{20}\text{Te}_{80}$ @ 400 C

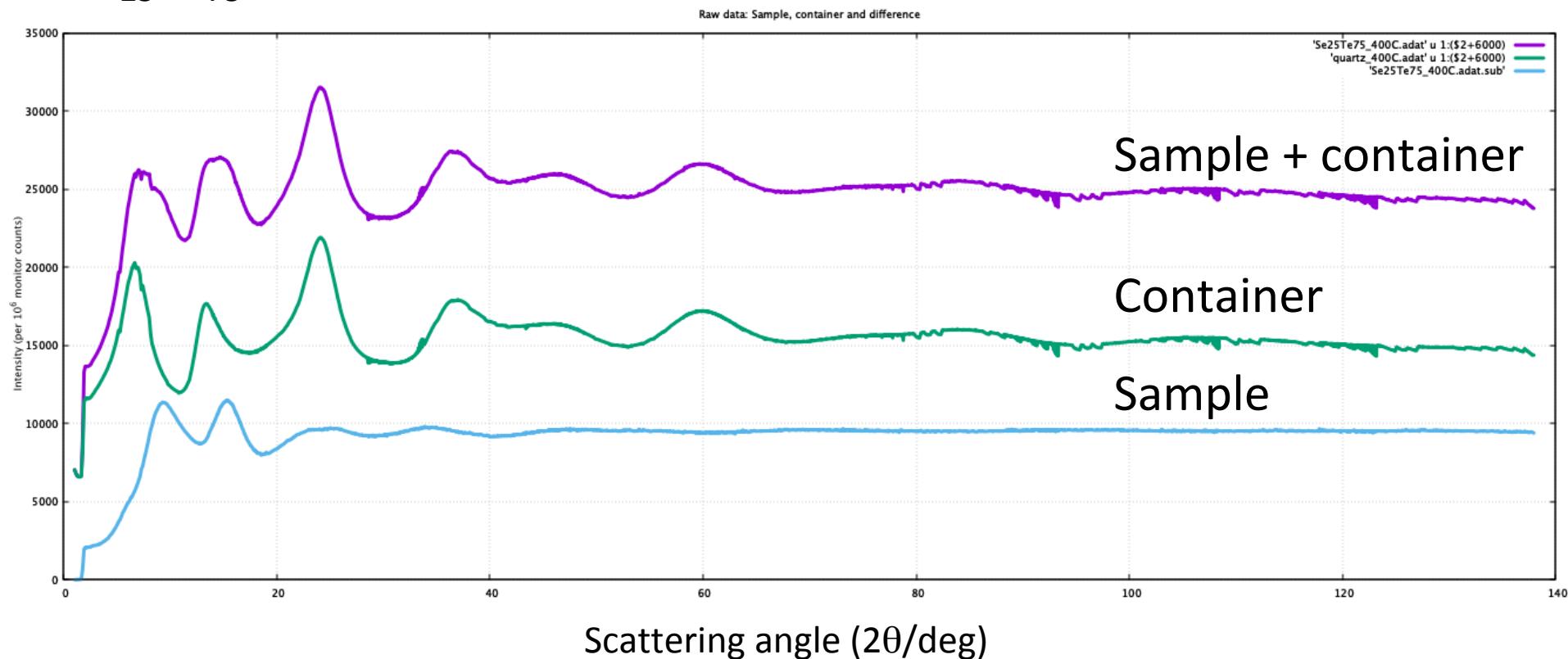


$$\text{Sample} = (\text{Sample+container}) - \text{Container}$$



Raw data

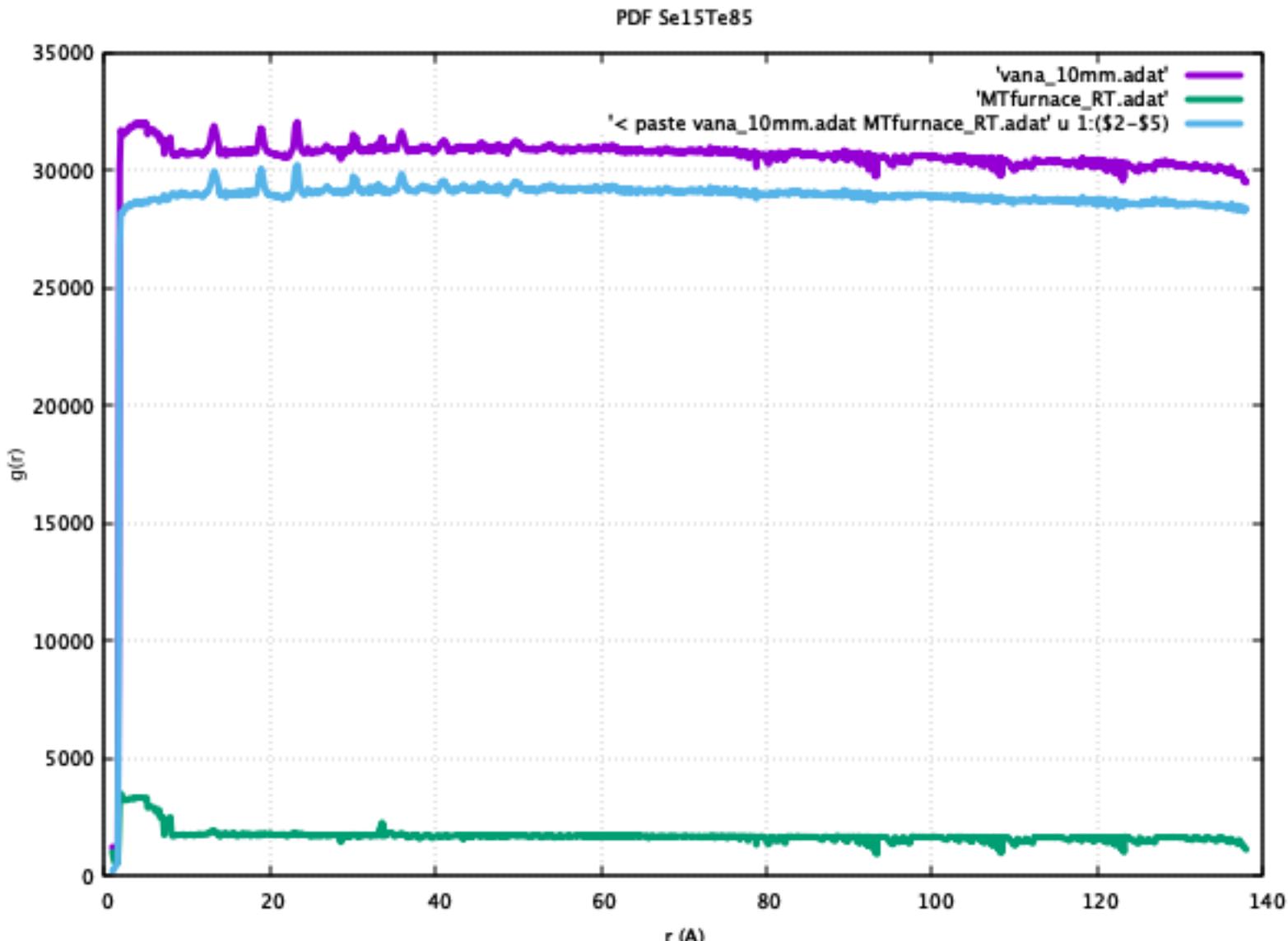
$\text{Se}_{25}\text{Te}_{75}$ @ 400 C



$$\text{Sample} = (\text{Sample+container}) - \text{Container}$$



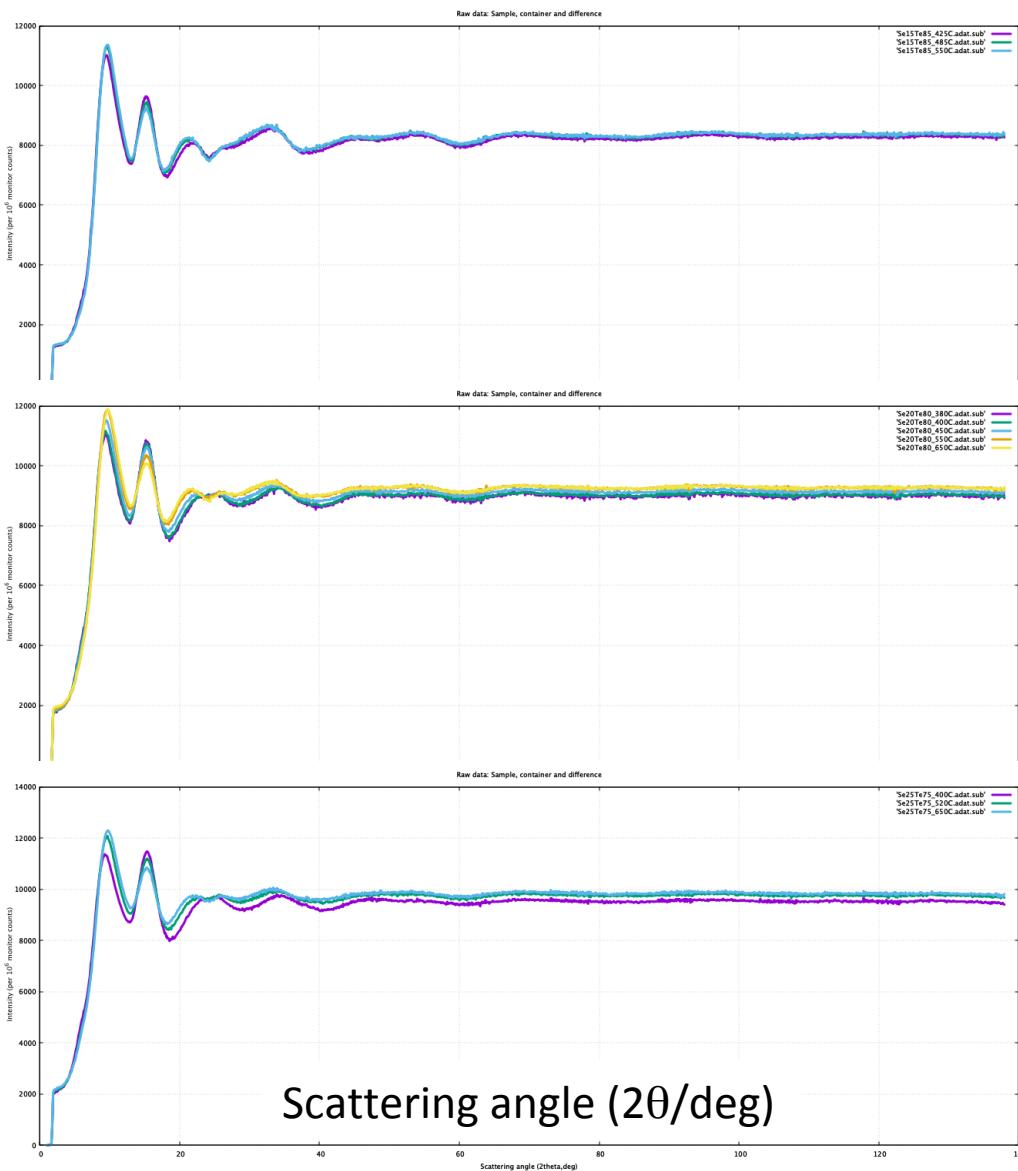
Raw data





Raw data

Intensity (10⁶ monitor counts)



$\text{Se}_{15}\text{Te}_{85}$

$\text{Se}_{20}\text{Te}_{80}$

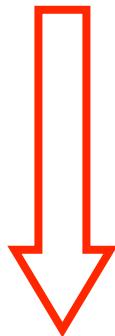
$\text{Se}_{25}\text{Te}_{75}$



Data reduction

$$I(2\theta, \omega) = C \Phi_0 N \frac{k'}{k} \frac{\sigma}{4\pi} S(\vec{Q}, \omega) \epsilon(k')$$

$$I(2\theta) = C \Phi_0 N \frac{\sigma}{4\pi} \int_{-\infty}^{E_{\max}} d\omega \frac{k'}{k} S(\vec{Q}, \omega) \epsilon(k')$$



Formal aspects

- Elastic scattering (diffraction)
 - Stationary beam
 - Constant efficiency detector
 - One interaction processes (single scattering)
- Bragg's law (for Q)
Integration limits ($\pm\infty$)

$$I(2\theta) = C \Phi_0 N / 4\pi (\sigma_{\text{coh}} S(Q) + \sigma_{\text{inc}}) \epsilon(k)$$

Practical aspects

- Monochromatic beam
- No background
- No attenuation
- Single scattering



- No beam
- No container
- No sample
- No environment
- No detector



No problem!



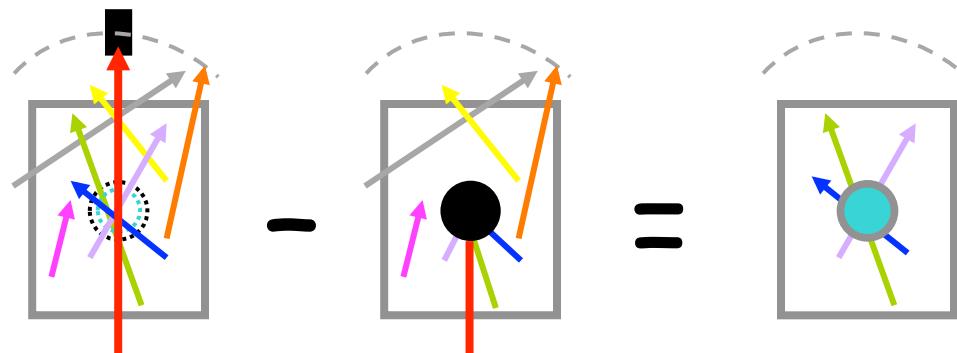
Experimental corrections

Instrumental effects

- Background
- Detector efficiency
- Detector dead-time
- Instrumental resolution

Sample effects

- Inelasticity
- Attenuation (container)
- Multiple scattering
- Normalisation

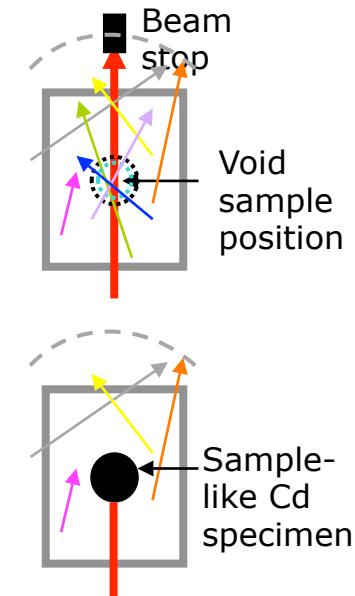


Background noise

Requires two measurements:

- Empty beam
(no sample, no container)

- Sample-like Cd specimen



$$T = \exp\{-n \sigma_T(E) d\}$$

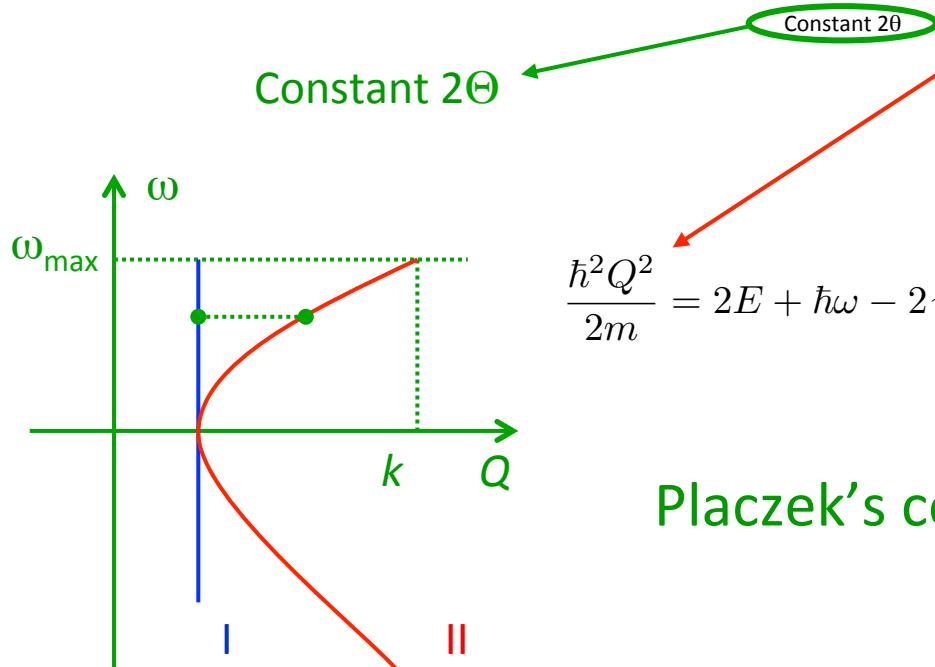
Inelasticity effects



$$I(2\theta) = C \Phi_0 N \frac{\sigma}{4\pi} \int_{-\infty}^{\infty} E_{max} \frac{k'}{k} S(\vec{Q}, \omega) \epsilon(k') d\omega$$

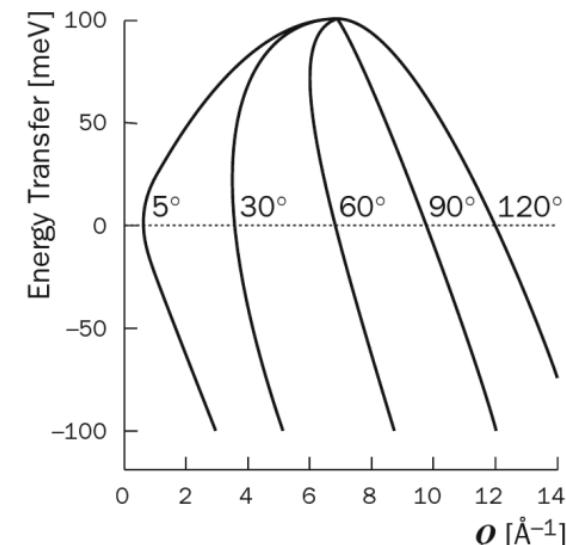
1

$\epsilon(k)$



$$\frac{\hbar^2 Q^2}{2m} = 2E + \hbar\omega - 2\sqrt{E^2 + \hbar\omega E} \cos 2\theta$$

Placzek's correction



These effects are closely associated
to the detector efficiency



Placzek's corrections

Efficiency

- Black detector, $\varepsilon(E) = 1$
- $1/v$ detector, $\varepsilon(E) \propto E^{-1/2}$
- Exponential detector, $\varepsilon(E) = 1 - \exp\{-\alpha (E/E')^{1/2}\}$

- Taylor expansion of $S(Q_i, \omega)$ around $(Q_i, \omega) \longrightarrow S(Q_{||}, \omega)$
- Expansion of $Q_{||}^2 - Q_i^2$, $\varepsilon(k)$ and k'/k in powers of ω/ω_{\max}
- Energy integration

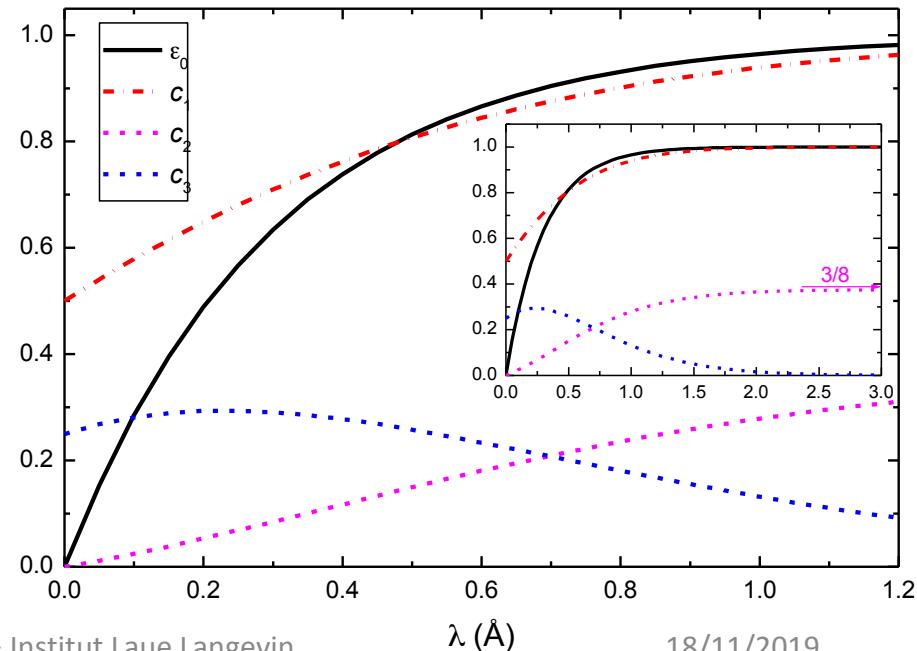
$$S(Q) = \frac{1}{\epsilon_0} \frac{1}{b_{coh}^2} \left(\frac{d\sigma}{d\Omega} \right)_{corr} + \left(1 + \frac{b_{inc}^2}{b_{coh}^2} \right) \left(C_1 \delta - C_2 \delta^2 + C_3 \delta \gamma - \frac{m}{2M} (\delta + \gamma) \right) - \frac{b_{inc}^2}{b_{coh}^2}$$

For an exponential detector

$$C_1 = 1 - \frac{\alpha/2}{e^\alpha - 1}$$

$$C_2 = \frac{3}{8} - \frac{\alpha(\alpha+3)}{8(e^\alpha - 1)}$$

$$C_3 = \frac{\alpha(\alpha+1)}{4(e^\alpha - 1)}$$





Sample related corrections

Attenuation

Multiple Scattering

Sample
+
Container

Minimisation by choosing an adequate sample geometry

$$I_S^{\text{corr}}(2\theta) = \frac{1}{\alpha_{S,SC}(2\theta)} \left(I_S(2\theta) - I_S^B(2\theta) - \frac{\alpha_{C,SC}(2\theta)}{\alpha_{C,C}(2\theta)} (I_C(2\theta) - I_C^B(2\theta)) \right) - \Delta$$

Paalman & Pings' coefficients

Cylindrical geometry

Blech & Averbach's correction

Complete knowledge of $S(\mathbf{Q}, \omega)$
Numerical simulation

Normalisation

Absolute scale
Vanadium diffractogram



Correction programs

Required data

CORRECT, Gudrun,
etc. etc.

- Sample + container : as many as (sample, T) pairs.
- Container : enough to be able to interpolate.
- Empty instrument : the sample chamber and the sample environment.
- Absorber : a sample-like absorber
(Cd for thermal neutrons and B for hot neutrons).
- Vanadium rod : a sample-like vanadium.
- Crystalline powder sample : for wavelength and zero-angle (Ni).
- A good description of the geometry.

Output: Differential cross section in barns/sterad./scatt. unit

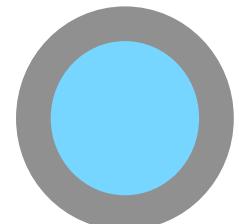
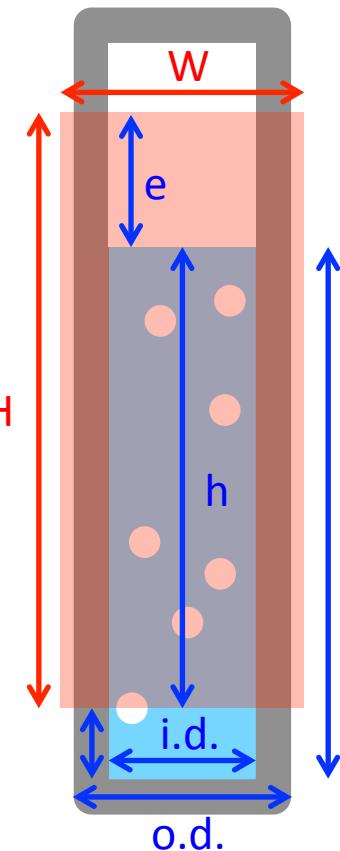
$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{corr}}$$



Input file for CORRECT

```
! Se15Te85_425C.com
!
inst D4
!
sample "Se15Te85_425C.adat" 0.45 /temperature=698.0 /density=0.026857 /packing=0.95668
! ds/d0_coh_self per atom = avr(bcoh^2) : 0.38122 barns/sr
! total sample composition in relative fractions (integer number of atoms/10)
component 1.5 Se
component 8.5 Te
!
! Container quartz o.d.= 11 mm, i.d.= 9 mm. Wall thickness = 1 mm
container "quartz_425C_110.adat" 0.55
!
! The background is the empty furnace
background "MTfurnace_RT.adat"
!
! 6mm08 vanadium with mtfurnace background
vanadium "vana_10mm.adat" 0.5 /smoothing=1
background /vanadium "MTfurnace_RT.adat"
!
wavelength 0.4989
! zeroangle = -0.090 already subtracted
zeroangle 0.0
! Beam dimensions in cm (height x width)
beam 5.0 1.2
! Placzek correction (inelasticity)
placzek SERIES_EXPANSION
!
! The output file is the coherent differential cross section
! corrected by inelasticity, multiple scattering, attenuation
! and properly normalised by vanadium
xout q
output "Se15Te85_425C.corr.q"
spectrum 1
execute/nopause
!
quit
```

$$\text{packing} = \rho_{\text{eff}} / \rho$$
$$\text{fullness} = h / H$$





Output CORRECT (1)

Check the input data

```
CORRECT version ILL 3.20 (3 July 17)
CORRECT> @Se15Te85_425C
CORRECT> ! Se15Te85_425C.com
CORRECT> !
CORRECT> inst D4
CORRECT> !
CORRECT> sample "Se15Te85_425C.adat" 0.45 /temperature=698.0 /density=0.026857 /packing=0.95668
CORRECT> ! ds/d0_coh_self per atom = avr(bcoh^2) : 0.38122 barns/sr
CORRECT> ! total sample composition in relative fractions:
CORRECT> component 0.15 Se
CORRECT> component 0.85 Te
CORRECT> container "quartz_425C_110.adat" 0.55
CORRECT> background "MTfurnace_RT.adat"
CORRECT> !
CORRECT> ! 6mm08 vanadium with mtfurnace background
CORRECT> vanadium "vana_10mm.adat" 0.5 /smoothing=1
CORRECT> background /vanadium "MTfurnace_RT.adat"
CORRECT> !
CORRECT> wavelength 0.4989
CORRECT> ! zeroangle = -0.090 already subtracted
CORRECT> zeroangle 0.0
CORRECT> beam 5.0 1.2
CORRECT> placzek SERIES_EXPANSION
CORRECT> !
CORRECT> !xout angle
CORRECT> !output aa_1212.corr
CORRECT> !title "filename.corr (after correct)"
CORRECT> !spectrum 1
CORRECT> !execute/nopause
CORRECT> !
CORRECT> xout q
CORRECT> output "Se15Te85_425C.corr.q"
CORRECT> spectrum 1
CORRECT> execute/nopause
```



Output CORRECT (2)

General information

CORRECT

=====

Correcting data from 2-AXIS diffractometer D4

Sample data is read from file Se15Te85_425C.adat
with background from file MTfurnace_RT.adat

Container data is read from file quartz_425C_110.adat
with background from file MTfurnace_RT.adat

Vanadium data is read from file vana_10mm.adat
with background from file MTfurnace_RT.adat
Vanadium smoothing with polynomial order 0

Output will be to file Se15Te85_425C.corr.q

Can absorption c/s (at 2200m/s) : 5.080 barns
Can scattering c/s : 4.950 barns
Can number density : 0.0722 per cu. Angstrom
V absorption c/s (at 2200m/s) : 5.080 barns
Vanadium scattering c/s : 4.950 barns
Vanadium number density : 0.0722 per cu. Angstrom
Sample intrinsic number density : 0.0269 per cu. Angstrom
Sample packing fraction : 0.9567
Sample fullness : 1.0000
Sample temperature : 698.0 K
Sample new title : filename.corr.q (after correct)

Sample geometry is CYLINDRICAL

Sample radius	: 0.4500 cm
Can outer radius	: 0.5500 cm
Vanadium radius	: 0.5000 cm

The sample consists of 2 species:

at.num	symbol	rel.conc.	at.weight	bcoh	scatt.c-s	incoh.c-s	abs.c-s
34	Se	0.15000	78.9600	7.9700	8.3000	0.3177	11.7000
52	Te	0.85000	127.6000	5.8000	4.3200	0.0927	4.7000

Beam height : 5.0000 cm
Beam width : 1.2000 cm

Placzek correction will be SERIES_EXPANSION

Incident wavelength : 0.4989 Angstroms
2theta zeroangle : 0.0000 Degrees

1 spectra will be corrected:

Spectrum 1

For the vanadium at wavelength 0.49890 AA :

mu_scat = scattering attenuation constant : 0.35739 cm⁻¹
mu_abs = absorption attenuation constant : 0.10177 cm⁻¹
mu_tot = total attenuation constant : 0.45916 cm⁻¹

And using an effective pathlength = $\sqrt{\pi} \cdot r \cdot 0.85$ of 0.75329 cm :
scattering of beam (w/o absorption) : 0.23602
true absorption of beam (w/o scattering) : 0.07380
transmission (i.e. 1 - total_attenuation) : 0.70760



Output CORRECT (3)

Some differential cross sections, etc, for the sample:

```
ds/d0_self per atom = avr(b^2)      : 0.39128 barns/sr
ds/d0_coh_self per atom = avr(bcoh^2) : 0.38122 barns/sr
ds/d0_incoh per atom = avr(bincoh^2) : 0.01006 barns/sr
--> fraction incoh/self : 0.02571
Yarnell ds/d0_self/atom=avr(b^2)      : 0.39161 barns/sr
Yarnell ds/d0_coh_self/atom=avr(bcoh^2): 0.38154 barns/sr
Yarnell ds/d0_incoh/atom=avr(bincoh^2) : 0.01007 barns/sr
--> Yarnell fraction incoh/self : 0.02571
```

```
b_coh = avr(b) over all isotopes in sample : 6.12550 fm
(avr(b))^2 over all isotopes in sample : 0.37522 barns/sr
And taking into account the imaginary part of b_coh:
b_coh = sample's avr(b) = 6.12550 fm + i * 0.00160 fm
|avr(b)|^2 over all isotopes in sample : 0.37522 barns/sr

avr(atwgt) over all isotopes in sample : 120.30400 a.u.
--> intrinsic mass density : 5.36444 g/cm**3
```

For the sample at wavelength 0.49890 AA :

```
mu_scat = scattering attenuation constant : 0.12634 cm-1
mu_abs = absorption attenuation constant : 0.04099 cm-1
mu_tot = total attenuation constant       : 0.16733 cm-1
```

And using an effective pathlength = $\sqrt{\pi} \cdot r \cdot 0.85$ of 0.67796 cm :

```
scattering of beam (w/o absorption)      : 0.08209
true absorption of beam (w/o scattering) : 0.02741
transmission (i.e. 1 - total_attenuation) : 0.89276
```

Cross sections

Sample bkg to be subtracted with coeff of 0.89276 which should correspond to the coefficient multiplying the MTcontainer scan in the background subtraction of the do_sample script file and should be roughly equal to the ratio Acsch/Acch evaluated at an intermediate angle.

Container bkg to be subtracted with coeff of 1.00000 which should correspond to (1 - the coefficient multiplying the e.g. MTbelljar scan in the background subtraction of the do_sample script file) and should be roughly equal to the Acch coefficient evaluated at an intermediate angle.

Furnace bkg to be subtracted with coeff of 1.00000 Vanadium bkg to be subtracted with coeff of 0.70760 which should correspond to the coefficient multiplying the e.g. MT_belljar scan in the background subtraction of the do_vanadium script file and should be slightly more than the Avvch coefficient evaluated at an intermediate angle.



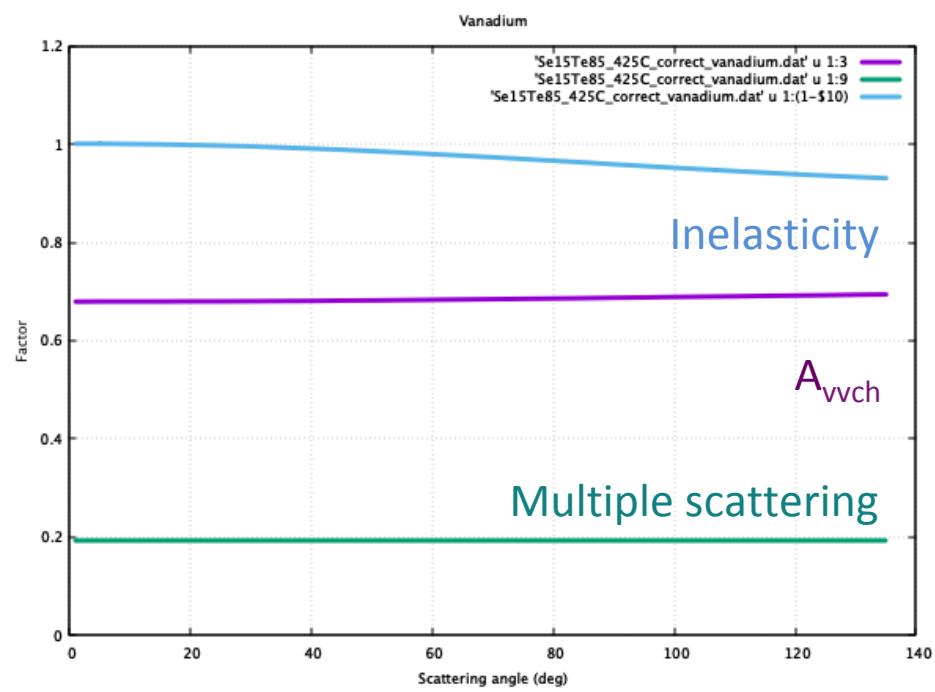
Output CORRECT (4)

Vanadium absorption, multiple scattering and Placzek corrections:

Linear attenuation coefficient (μ_{tot}) * radius = 0.22958

Angle	Q	Avvch	Acvch	Acch	Ahvch	Ahch	Ahh	m.s.	Placzek
1.06	0.23	0.67941	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	-0.00094
5.06	1.11	0.67944	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	-0.00078
10.06	2.21	0.67952	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	-0.00029
15.01	3.29	0.67964	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00051
20.01	4.38	0.67982	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00164
25.01	5.45	0.68004	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00306
30.01	6.52	0.68032	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00478
35.11	7.60	0.68065	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00682
40.11	8.64	0.68102	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.00908
45.00	9.64	0.68144	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.01152
50.00	10.64	0.68191	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.01424
55.00	11.63	0.68243	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.01716
60.00	12.59	0.68300	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.02024
65.05	13.54	0.68361	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.02351
70.05	14.46	0.68426	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.02687
75.05	15.34	0.68494	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.03033
80.08	16.20	0.68566	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.03387
85.08	17.03	0.68640	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.03743
90.08	17.82	0.68716	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.04100
95.11	18.59	0.68794	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.04458
100.11	19.31	0.68872	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.04810
105.05	19.99	0.68950	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.05151
110.05	20.64	0.69029	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.05486
115.05	21.25	0.69106	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.05810
120.05	21.82	0.69181	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.06120
125.06	22.35	0.69253	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.06413
130.06	22.83	0.69322	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.06688
135.06	23.28	0.69386	1.00000	1.00000	1.00000	1.00000	1.00000	0.19384	0.06943

Vanadium



- Paalman & Pings' coefficients
- Multiple scattering (Blech & Averbach)
- Inelasticity (Placzek)

Output CORRECT (4)

Sample absorption, multiple scattering and Placzek corrections:

Linear attenuation coefficient (μ_{tot}) * radius = 0.07530

Angle	Q	Assch	Acsch	Acch	Ahsch	Ahh	m.s.	Placzek
1.06	0.23	0.79040	0.79431	0.86102	1.00000	1.00000	1.00000	0.12060 -0.00037
5.06	1.11	0.79040	0.79435	0.86110	1.00000	1.00000	1.00000	0.12060 -0.00034
10.06	2.21	0.79041	0.79444	0.86119	1.00000	1.00000	1.00000	0.12060 -0.00025
15.01	3.29	0.79043	0.79458	0.86129	1.00000	1.00000	1.00000	0.12060 -0.00012
20.01	4.38	0.79045	0.79476	0.86138	1.00000	1.00000	1.00000	0.12060 0.00008
25.01	5.45	0.79049	0.79497	0.86146	1.00000	1.00000	1.00000	0.12060 0.00032
30.01	6.52	0.79054	0.79522	0.86154	1.00000	1.00000	1.00000	0.12060 0.00061
35.11	7.60	0.79059	0.79550	0.86160	1.00000	1.00000	1.00000	0.12060 0.00096
40.11	8.64	0.79066	0.79581	0.86165	1.00000	1.00000	1.00000	0.12060 0.00135
45.00	9.64	0.79072	0.79612	0.86169	1.00000	1.00000	1.00000	0.12060 0.00177
50.00	10.64	0.79080	0.79647	0.86172	1.00000	1.00000	1.00000	0.12060 0.00224
55.00	11.63	0.79087	0.79683	0.86173	1.00000	1.00000	1.00000	0.12060 0.00274
60.00	12.59	0.79096	0.79720	0.86174	1.00000	1.00000	1.00000	0.12060 0.00327
65.05	13.54	0.79104	0.79759	0.86174	1.00000	1.00000	1.00000	0.12060 0.00384
70.05	14.46	0.79113	0.79798	0.86173	1.00000	1.00000	1.00000	0.12060 0.00442
75.05	15.34	0.79122	0.79838	0.86171	1.00000	1.00000	1.00000	0.12060 0.00502
80.08	16.20	0.79132	0.79878	0.86169	1.00000	1.00000	1.00000	0.12060 0.00564
85.08	17.03	0.79141	0.79918	0.86167	1.00000	1.00000	1.00000	0.12060 0.00626
90.08	17.82	0.79151	0.79959	0.86165	1.00000	1.00000	1.00000	0.12060 0.00689
95.11	18.59	0.79160	0.79999	0.86164	1.00000	1.00000	1.00000	0.12060 0.00752
100.11	19.31	0.79170	0.80039	0.86164	1.00000	1.00000	1.00000	0.12060 0.00813
105.05	19.99	0.79179	0.80078	0.86165	1.00000	1.00000	1.00000	0.12060 0.00873
110.05	20.64	0.79188	0.80117	0.86169	1.00000	1.00000	1.00000	0.12060 0.00933
115.05	21.25	0.79197	0.80155	0.86175	1.00000	1.00000	1.00000	0.12060 0.00990
120.05	21.82	0.79206	0.80193	0.86184	1.00000	1.00000	1.00000	0.12060 0.01045
125.06	22.35	0.79214	0.80230	0.86197	1.00000	1.00000	1.00000	0.12060 0.01097
130.06	22.83	0.79223	0.80267	0.86214	1.00000	1.00000	1.00000	0.12060 0.01146
135.06	23.28	0.79230	0.80302	0.86237	1.00000	1.00000	1.00000	0.12060 0.01191

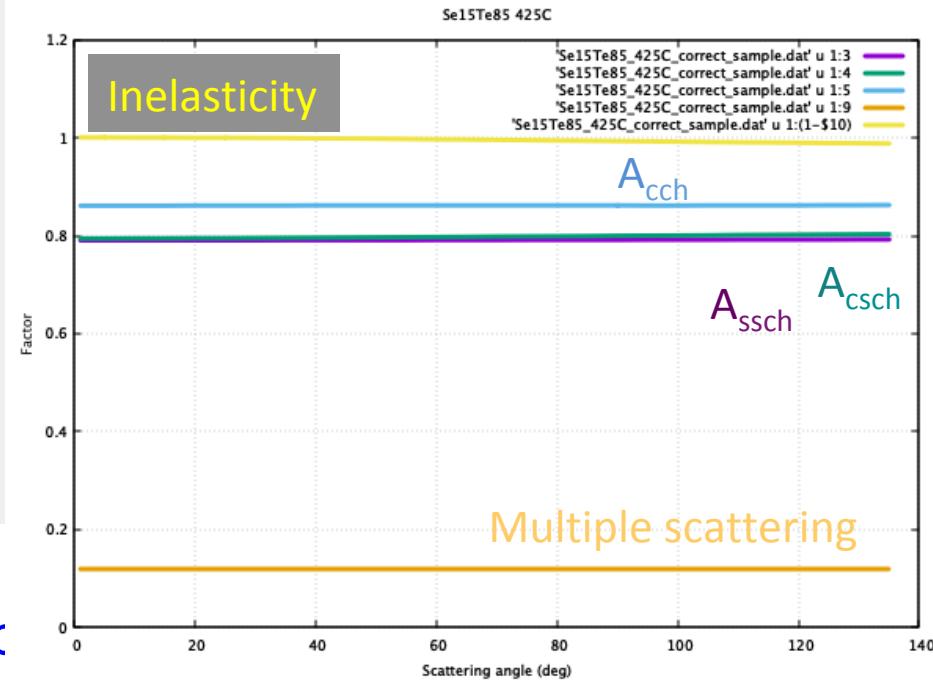
correct_2axis executed

CORRECT> !

CORRECT> quit

- Paalman & Pings' coefficients
- Multiple scattering (Blech & Averk)
- Inelasticity (Placzek)

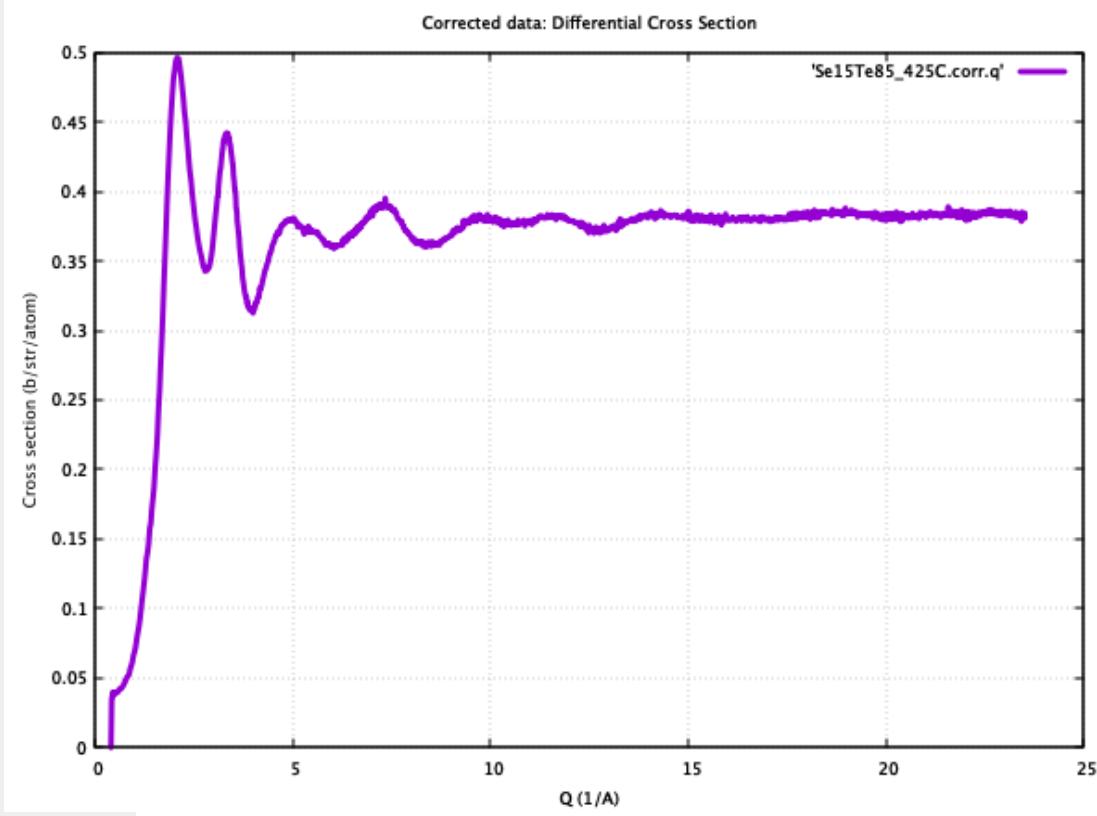
Sample





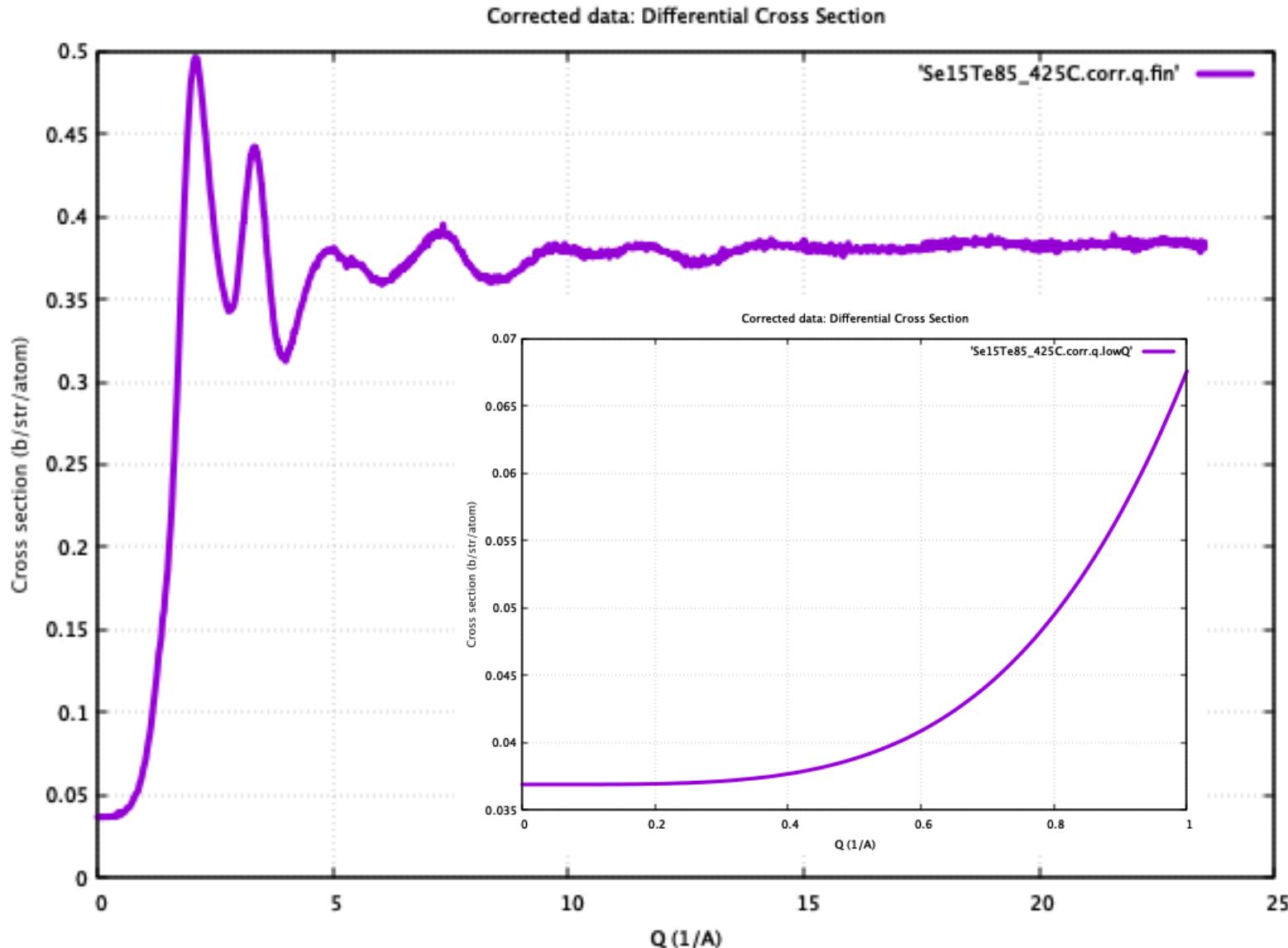
Output file CORRECT

```
# filename.corr.q (after correct)
#Block 1
=====
#
#Instrument: D4
#User      : cuello (/net/serdon/illdata/processed/163/d4/
exp_6-03-419/processed)
#Run number:          1
#Spectrum :          1
#Title     : filename.corr.q (after correct)
#Run date  : Mon Jan 30 14:34:39 2017
#X caption : Momentum transfer (A**-1)
#Y caption : Cross-section
#Histogram :          F
#Points    :          1448
 0.232993290 -4.62648682E-02  3.51537281E-04
 0.261566997 -4.64400165E-02  3.06130416E-04
 0.287942380 -4.63275239E-02  2.72539881E-04
 0.316515416 -4.66136783E-02  2.61351408E-04
 0.342890084 -4.60411571E-02  2.60503730E-04
 0.371462286 -4.54884954E-02  2.68991600E-04
 0.397836119 -1.95801519E-02  4.88558842E-04
 0.426407456  3.49235497E-02  7.47496902E-04
 0.452780336  3.98697816E-02  7.62447249E-04
 0.481350482  3.80599499E-02  7.57011061E-04
 0.507722378  3.90205234E-02  7.52640422E-04
 0.536291301  3.93197648E-02  7.49911123E-04
 0.562662005  3.99930254E-02  7.50510953E-04
 0.591229618  4.01250273E-02  7.55647954E-04
 0.617598951  4.20581698E-02  7.60504045E-04
 0.646165073  4.25601229E-02  7.65911187E-04
 0.672532976  4.25573699E-02  7.71594932E-04
 0.701097429  4.38850597E-02  7.76210974E-04
 0.727463782  4.49257679E-02  7.81205890E-04
 0.756026447  4.70207781E-02  7.87560362E-04
 0.782391071  4.85052131E-02  7.92242179E-04
```



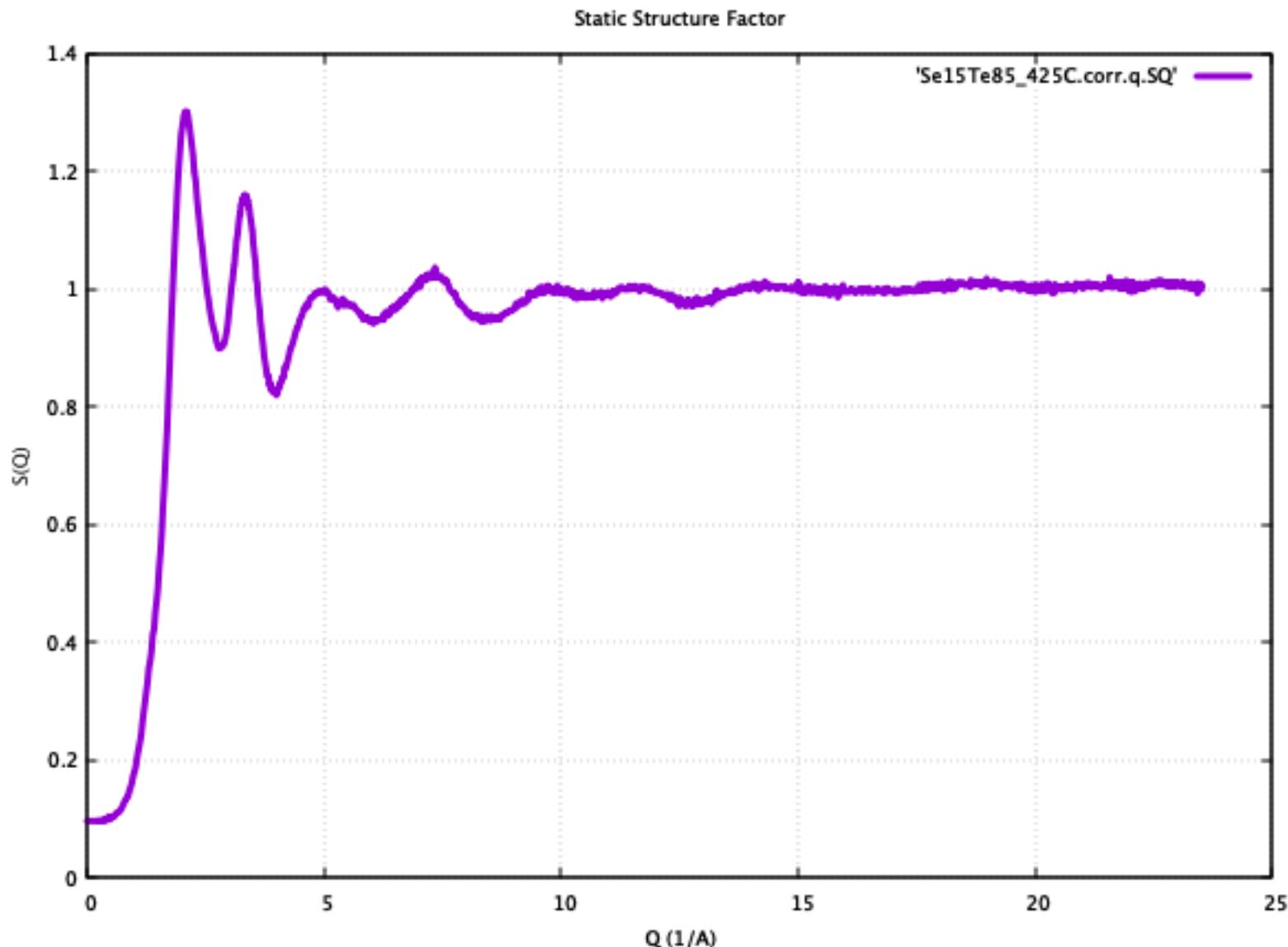


Extrapolation to low- Q



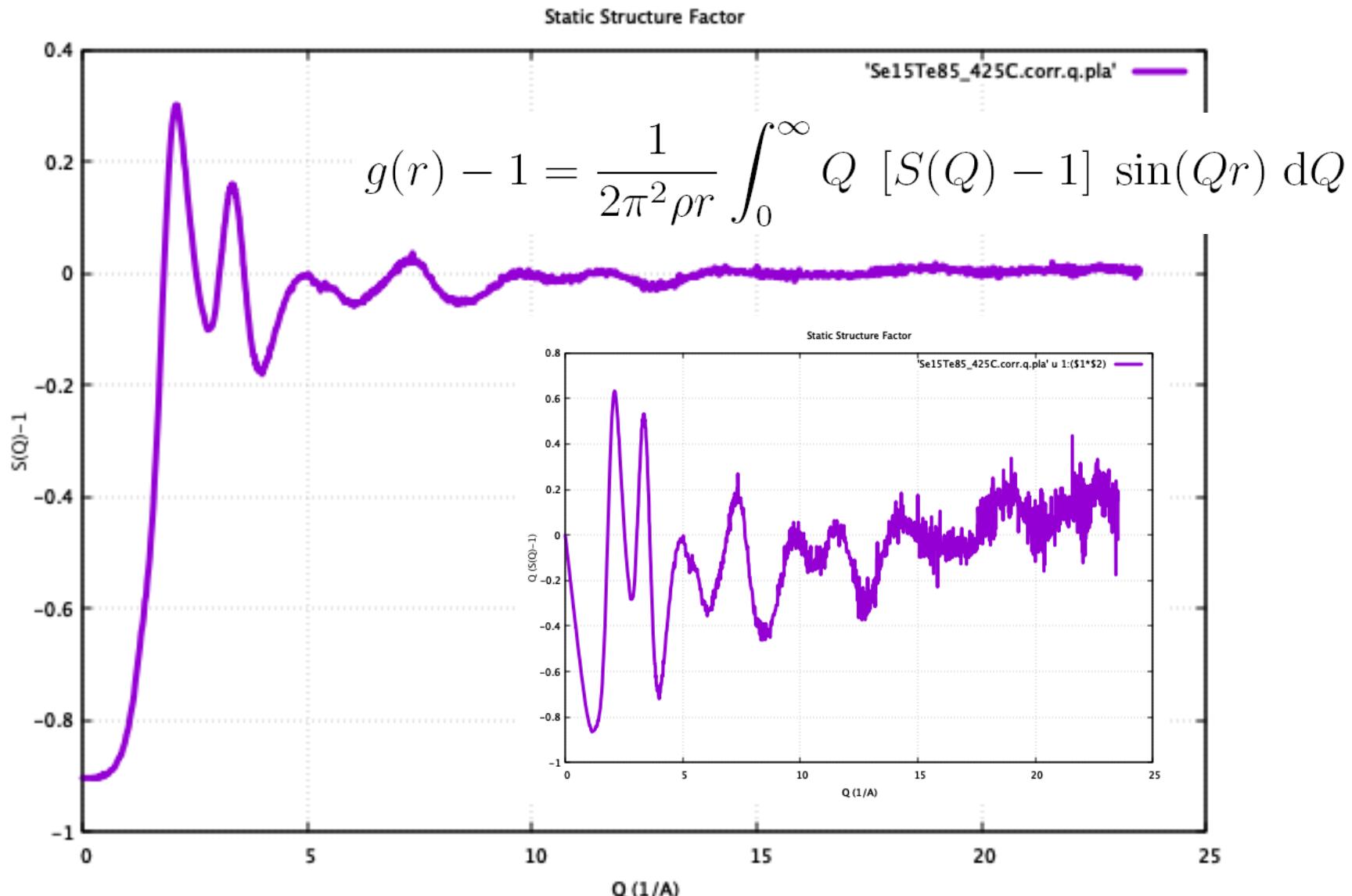


Static Structure factor $S(Q)$



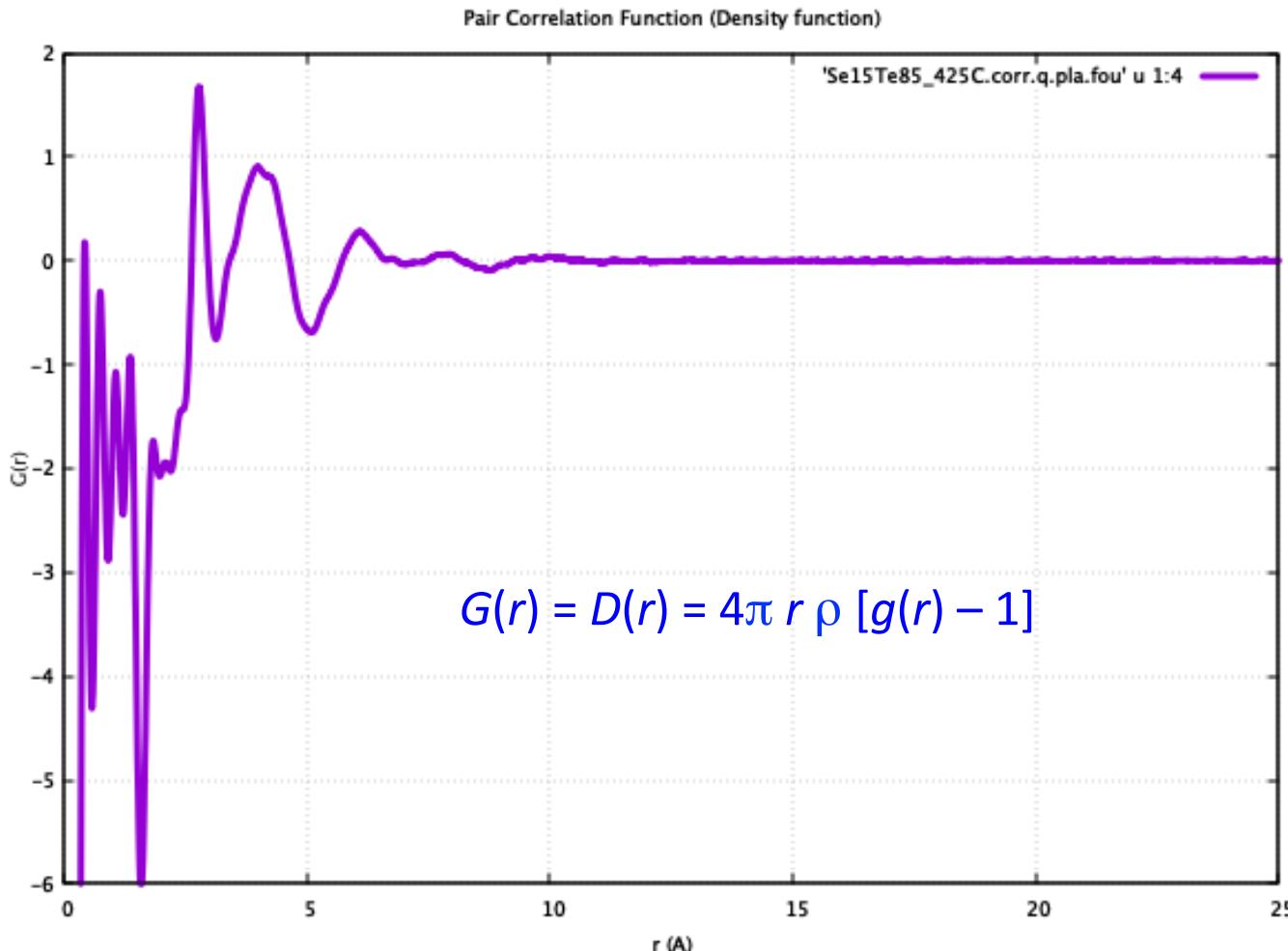


$$F(Q) = S(Q)-1$$



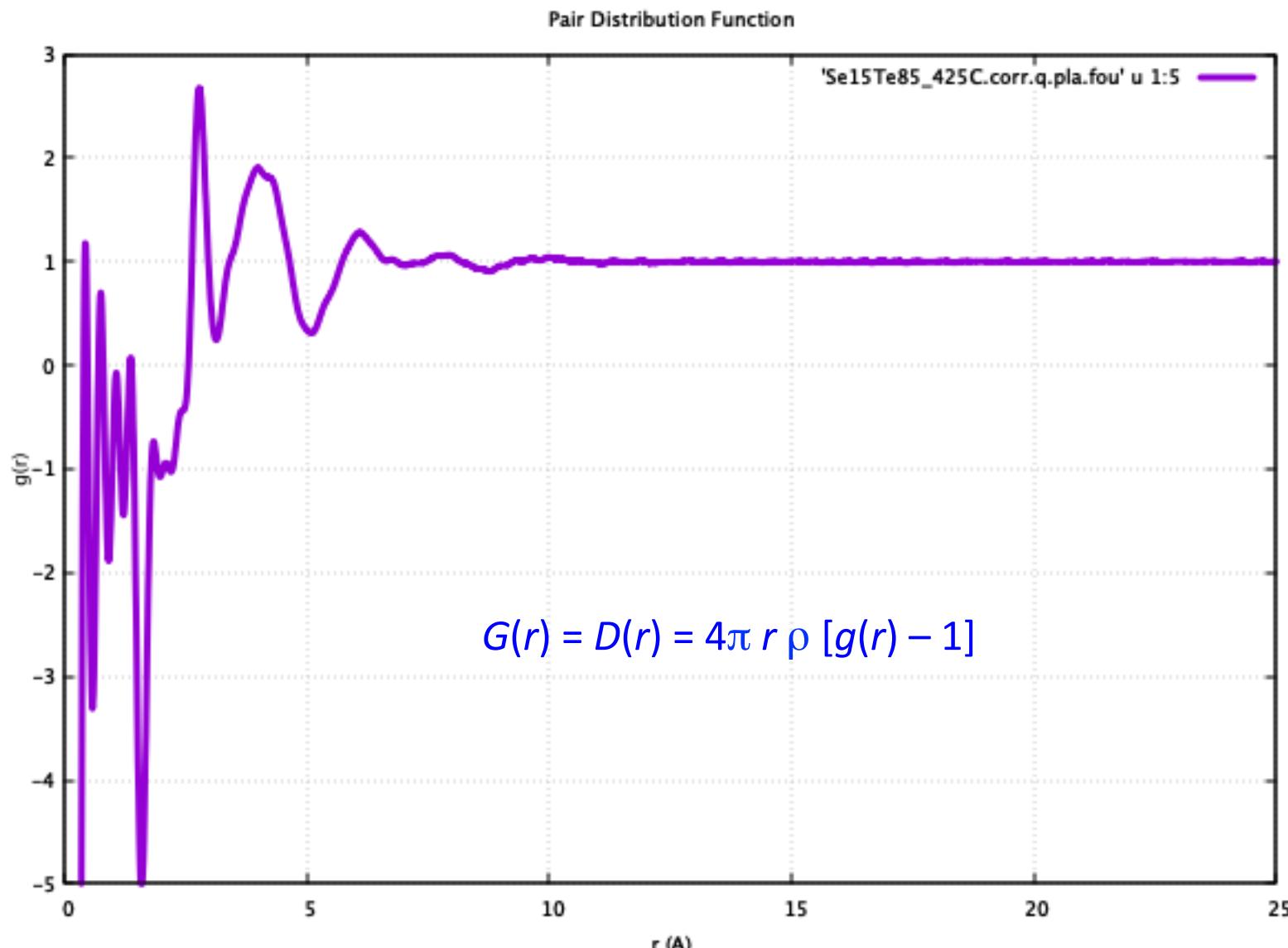
Sinus FT: Pair Correlation Function

$$g(r) - 1 = \frac{1}{2\pi^2 \rho r} \int_0^\infty Q [S(Q) - 1] \sin(Qr) dQ$$



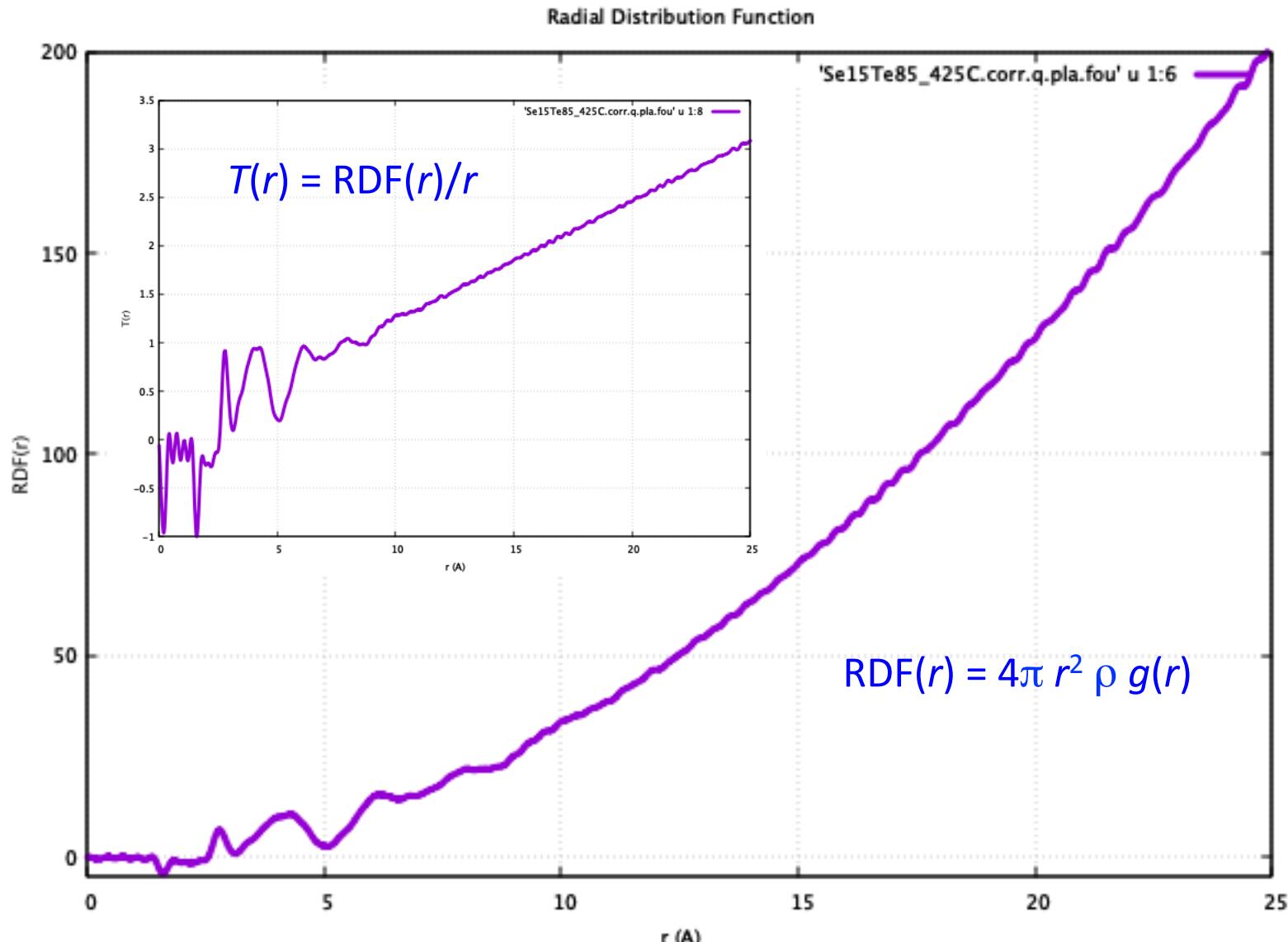


Pair Distribution Function (PDF)



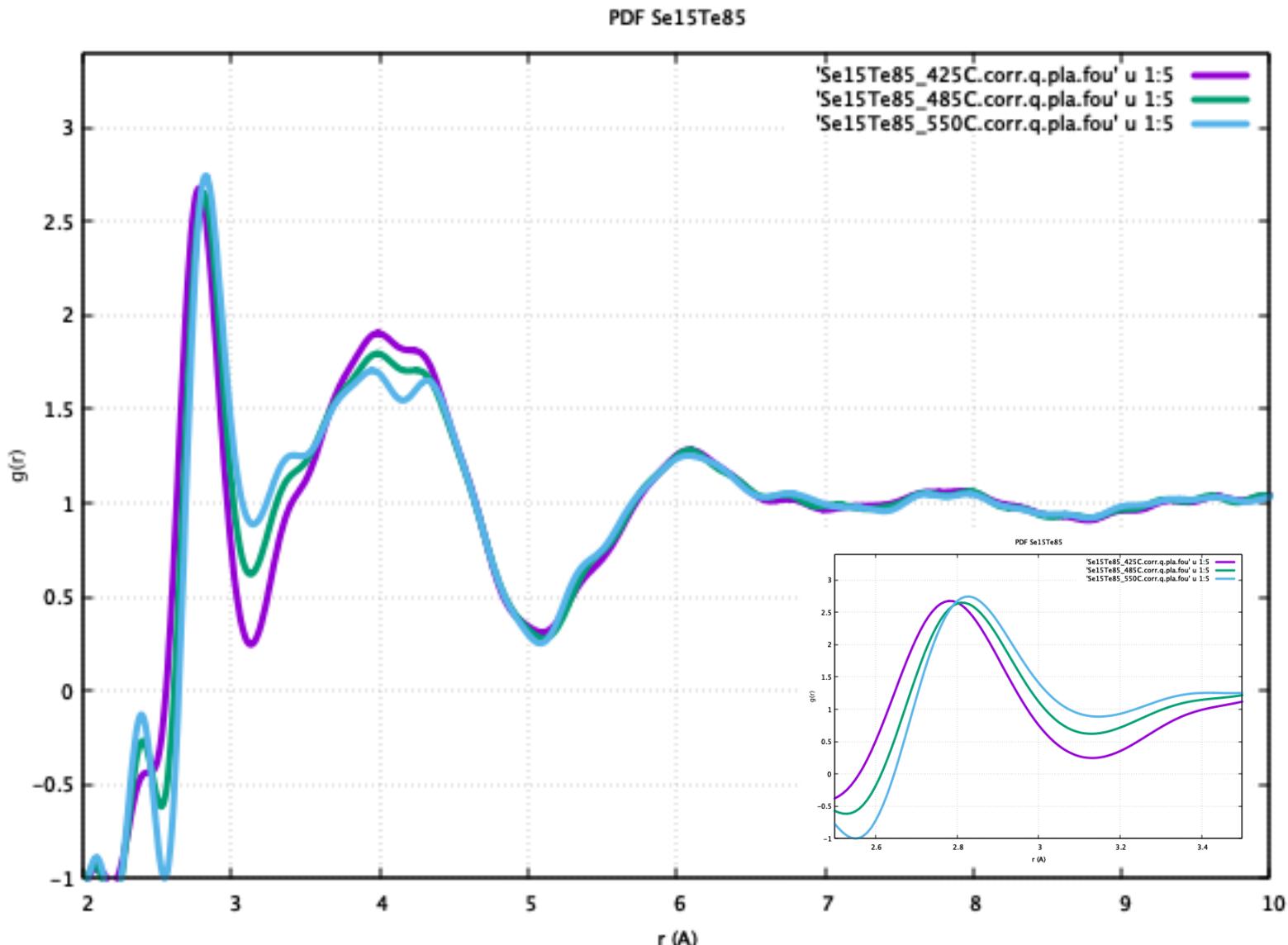


Radial Distribution Function

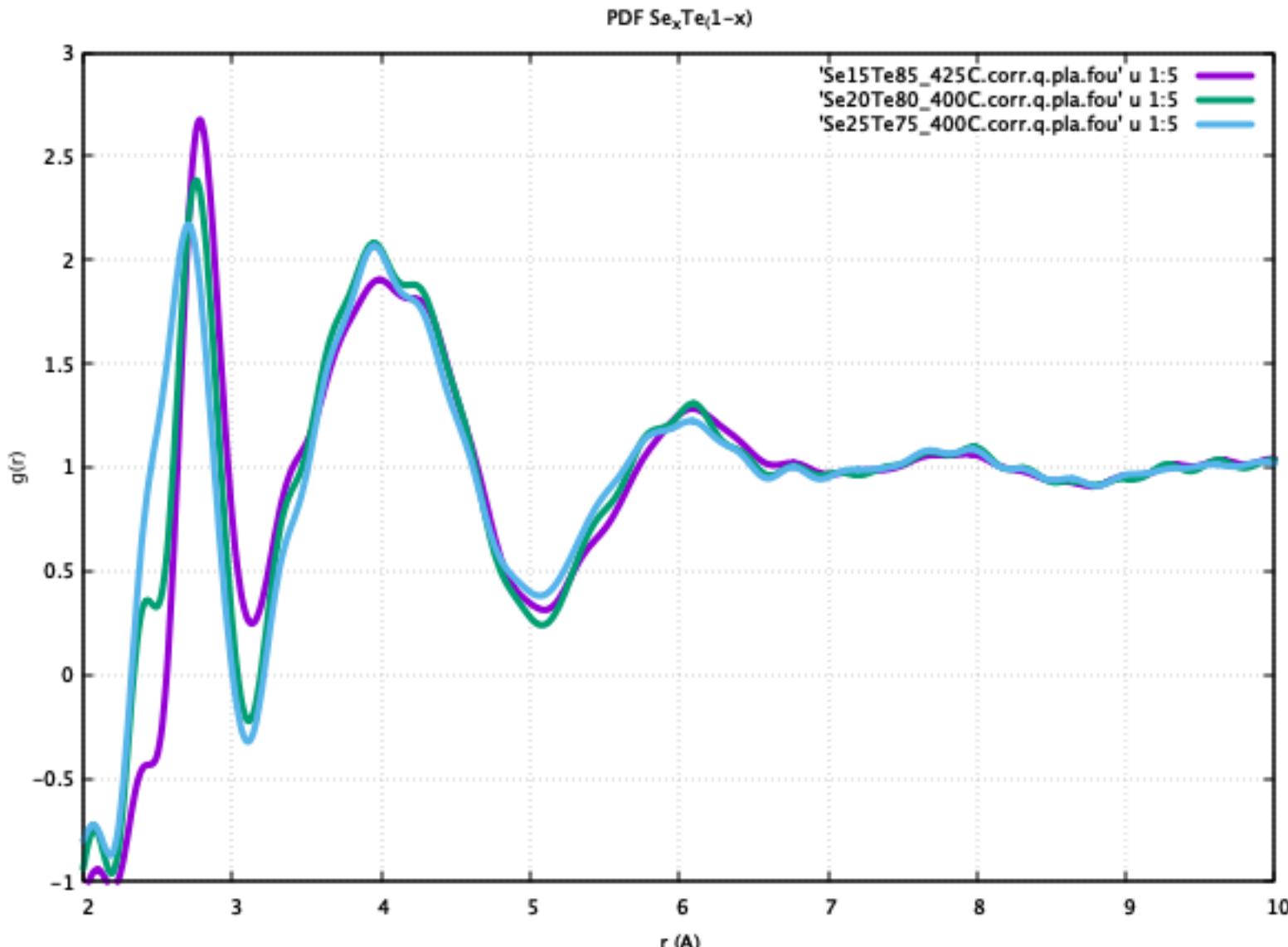




PDF $\text{Se}_{15}\text{Te}_{85}$

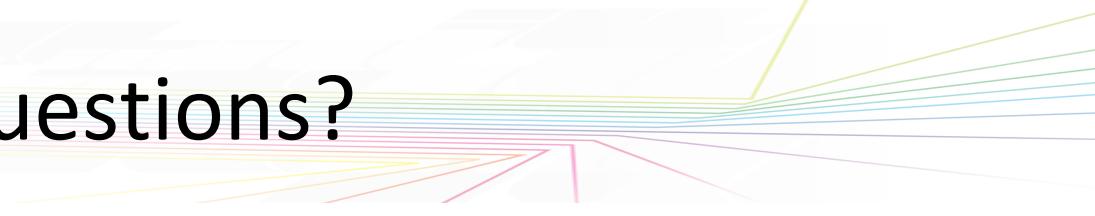


PDF $\text{Se}_x\text{Te}_{1-x}$





Questions?



Presentation and tutorial available at the School website

cuello@ill.fr